Structural Reliability
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Lecture –45
Common Probability Distributions (Part - 16)

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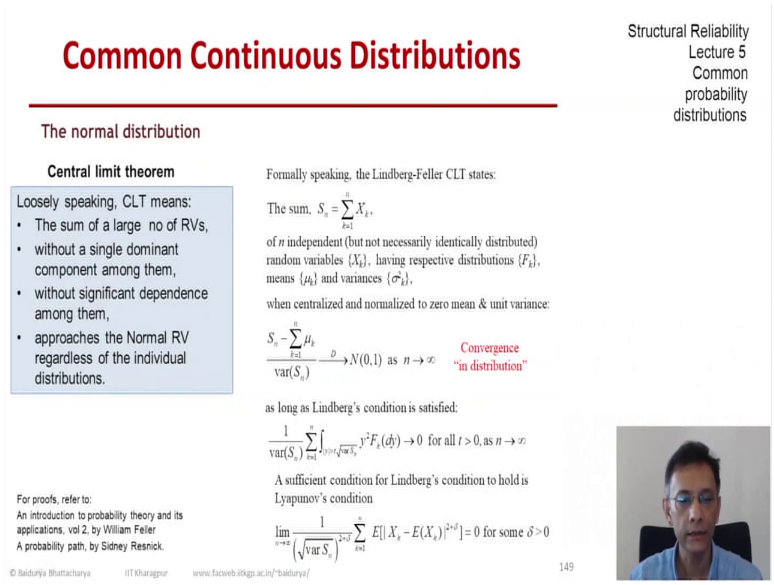

The normal distribution it is seemingly everywhere and that is because of the central limit theorem. So, loosely speaking the central limit theorem states that if you add a large number of random variables without a single dominant component among them and without significant dependence among them then that sum would approach the normal distribution regardless of the individual distribution.

So, that makes the central limit theorem very powerful. And it is also a very forgiving theorem it applies in many cases but one should be careful not to force in it into situations where it clearly does not apply. So, formally speaking the Lindbergh fellow version of CLT states that if you take the sum of n independent random variables not necessarily identically distributed with respect to distributions as you see the sequence F k the means the sequence mu k and the variances sigma square k.

And when you centralize the sum which means you take the means out and you normalize with the variance of the sum then that quantity approaches the normal distribution the standard normal distribution with mean 0 and variance 1. And as we will see later this convergence is in distribution. Now obviously it requires some conditions. So, this statement of the central limit theorem is satisfied.

As long as Lindbergh's condition is satisfied which you see on the screen which basically says that as you keep adding terms the probability of finding the probability mass further and further away from the mean is less and less likely for any of these random variables. And a sufficient condition for Lindbergh's condition is the Leoponov's condition and that is actually in many cases easier to verify.

If you want to see the formal proofs refer to the excellent texts the book by the bible by Feller and the book by Resnick that I have been referring to in these lectures.

Let us just in the next slide look at just one illustration of how quickly the central limit theorem kicks in.

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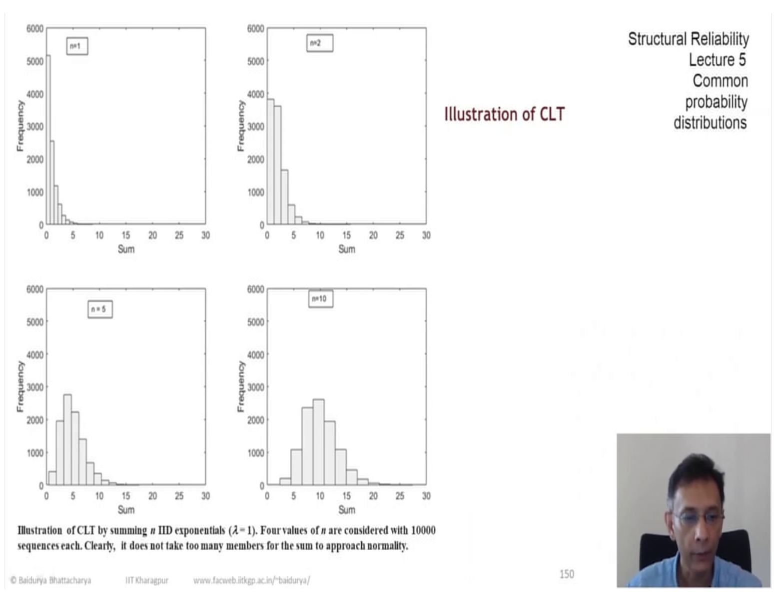

So, what you see here is on the on the top left corner you see the exponential random variable about 10 000 simulations of that and you can clearly see the exponential nature its mean is one and. Now if you just add two of these. So, the next figure on the right at the n equals 2 that is the distribution the histogram of the sum of these two independent exponentials with unit mean and you can see that the peak of the exponential has already been blunted.

So, that is just when you add two if you add five of them the exponential nature is almost gone and you can see that it is trying to reach that nice middle peak with tails on both sides. And if you just add 10 of them it does not have to be large in the literal sense even if you add 10 of them these independent exponentials you get a sum which looks clearer and clearer to have the nice bell-shaped Gaussian form.

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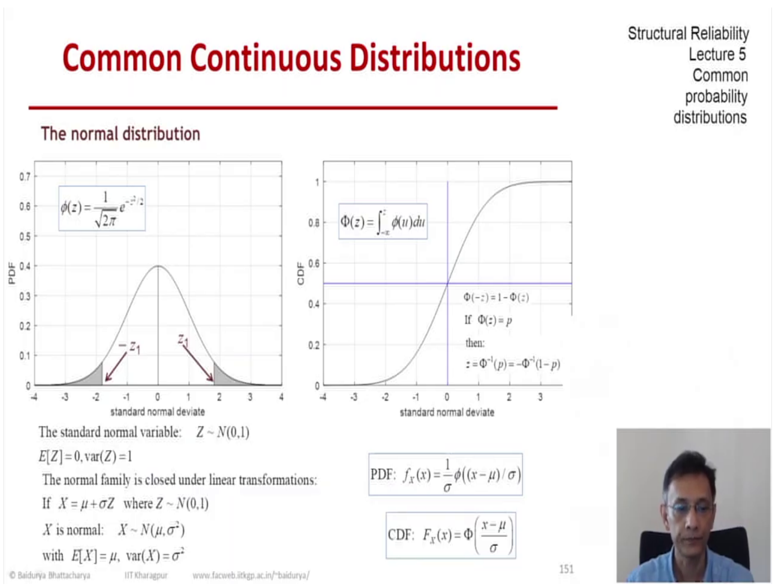

So the standard normal density function and the standard normal distribution functions are here on the screen and it is basically the error function and if you integrate it the area under it is 1 the mean is 0 and the variance is 1. So, that is the standard normal random variable. One of the important properties which comes from the symmetry of the distribution is the area to the right of z 1 is identical to the area to the left of negative z 1.

And this is used very conveniently when we try to find the normal distribution function evaluated at say a negative number. So, that is what you see on the top right figure that if phi of z is p then z is either phi inverse of p or negative phi inverse of 1 - p. Now one of the most powerful properties of the normal distribution are that the family is closed under linear transformation. So, if x is normal. So, is a x + b.

So, if I transform the standard normal with the help of mu and sigma that you see on the screen then I get a normal random variable whose mean is mu and standard deviation is sigma. So, that is actually what is used when we try to find the CDF of any arbitrary random variable because we can go back to the standard normal CDF.