Structural Reliability
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Lecture –44
Common Probability Distributions (Part - 15)

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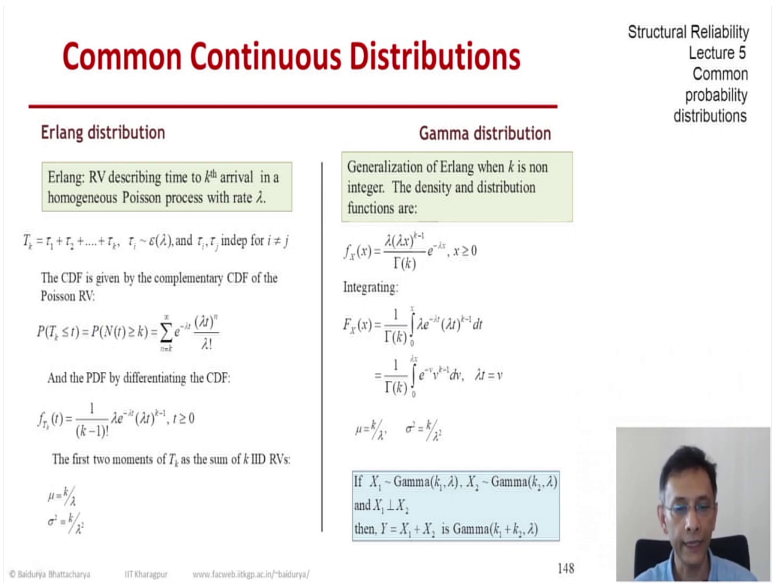

The online distribution describes the time to the k^{th} arrival in a homogeneous Poisson process with rate lambda. So, just as we had in the case of the sequence of Bernoulli's trials. The number of trials to the k^{th} occurrence the negative binomial random variable being the sum of k independent and identical geometric random variables we have the Erlang random variable as a sum of k independent and identical exponentials with the same rate lambda obviously.

So, that would give me the CDF of the Erlang distributions the same way as we derived it for the negative binomial case from the Poisson CDF and differentiating that with respect to time we get the Erlang PDF. Going back to the first equation that you see on the screen the mean and the variance of T k can be easily found as the mean of the sum of those tau's and the variance of the sum of those tau's as k over lambda and k over lambda squared respectively.

Now what if k is not an integer we can generalize that k - 1 factorial in the PDF of T k and define the gamma distribution. So instead of k - 1 factorial we have gamma k in the denominator and we can integrate that density function to get the CDF which again involves the gamma function and it has the same mean and variance as in the case of the Erlang with the same k and lambda.

One interesting property that would come to be useful in some cases is that if you sum two independent gammas with the same lambda then their sum is again gamma with the same lambda and the k are given in terms of k 1 + k 2.