Structural Reliability
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Lecture –42
Common Probability Distributions (Part - 13)

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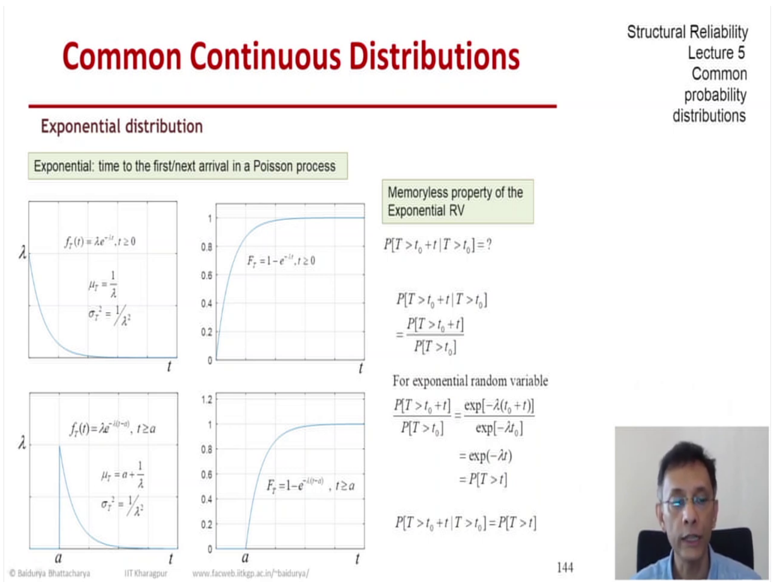

The exponential distribution the time to the first or the next occurrence in a Poisson process its density function is the well-known exponential decay that you see on the screen. And the CDF reaches 1 asymptotically. The mean is 1 over lambda the lambda being the rate parameter for the underlying Poisson process and the standard deviation is also one over lambda. In some situations uh it is useful to have the shifted exponential distribution which is what you see on the bottom part of the screen instead of 0 the distribution starts from the positive number a.

And the mean is also shifted accordingly so, it is a over 1 by lambda and the variance remains unchanged. Now you will hear that the exponential distribution has the famous memory less property. So, let us just see what this means and just to derive that. So, let us say that the random variable capital T is at least greater than t naught. So, I am waiting for something and I have waited at least up to t naught.

And given that fact I want to know how much longer do I have to wait. So, given t greater than t naught what is the probability that it will still be greater than t naught + t. So, for any distribution this conditional probability would be the ratio of the two probabilities that you see on the screen and for the exponential random variable now plugging in the exponential functional forms we arrive at the ratio that you see on the screen and that would end up as exponential minus lambda t which is nothing but P of T greater than t.

So, this is the memory less property of the exponential that given that I have weighted up to t naught I have to wait another length of interval t is the same as if I just started from zero. So, the process forgets about its past and just resets the clock.

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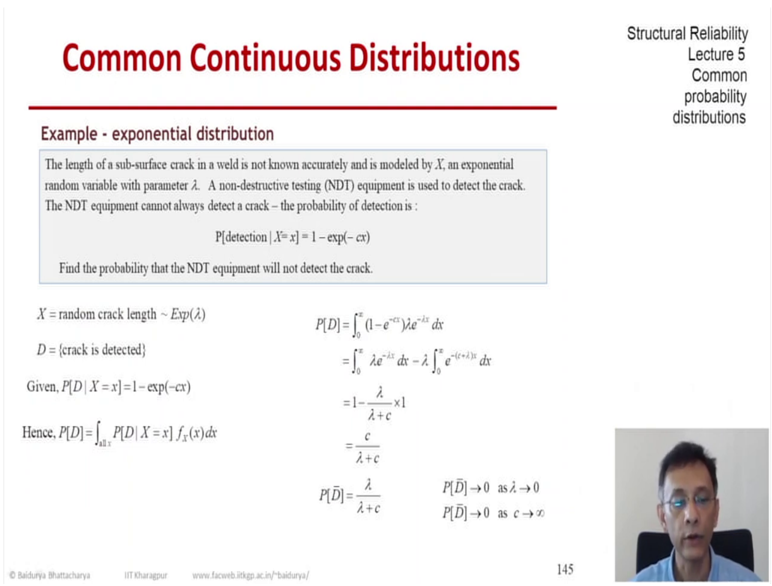

Let us move on to one more example involving the exponential and obviously the application of the exponential distribution goes beyond the Poisson process and the waiting time. So, here what you see is an example involving the line so, its length. So, the length of a crack is modeled by an exponential random variable x and there is a detection event which now depends on the size of the crack itself.

So, let us take a minute to read the problem and then start solving it. So, our random variable x is the exponential random variable with parameter lambda and d is the event of crack detection and we know what the conditional probability of d is given that the random variable x takes on a certain value. So, the probability of detection would be obtained through the use of total probability the integration that you see on the screen and plugging in the functional forms we can do the integration and the answer if you walk through would be c over lambda + c.

So, that is the probability of detection what is been asked for is it will not detect the crack. So, p of d bar would be lambda over lambda + c. So, let us look at the limiting cases and see if this answer makes sense. So, if the mean crack becomes large. So, the crack is huge in that case the detection probability should reach zero and that is exactly what we see as lambda goes to zero which means the mean becomes larger and larger I am almost certain to detect the crack likewise if my instrument becomes more and more sensitive.

So, c becomes larger and larger then also I am more and more likely to detect the crack. So, p of d bar would reach zero.