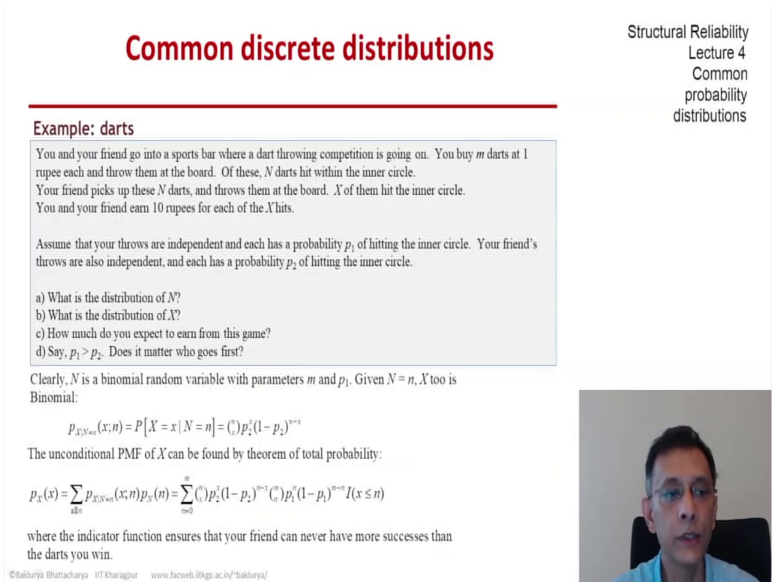
Structural Reliability
Prof. Baidurya Bhattacharya
Department of Civil Engineering
Indian Institute of Technology, Kharagpur

Lecture –38
Common Probability Distributions (Part - 09)

(Refer Slide Time: 00:28)



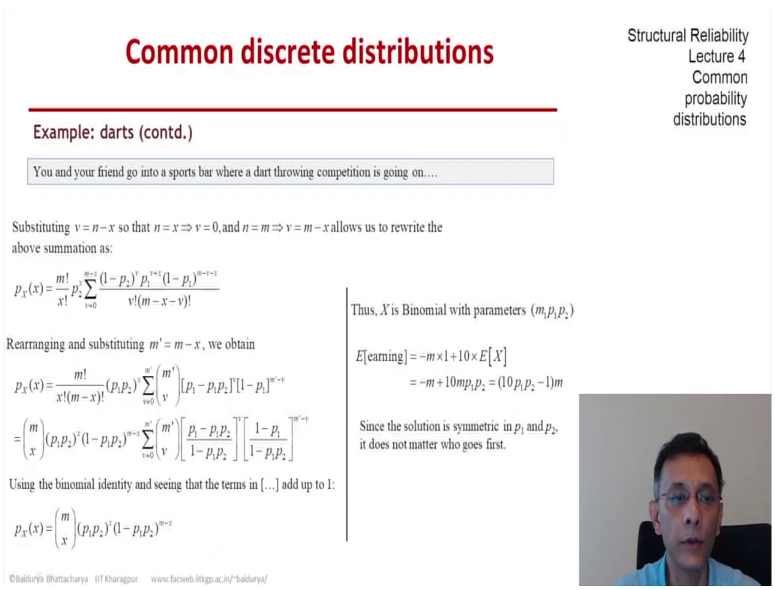
Here we look at a two-stage problem involving Bernoulli trials. So, let us take a minute reading the problem and then we will proceed. So, this is how the problem proceeds you start with a known number of dots trials m. So, lowercase m and then from there you filter out N capital N successes and you do a filtering once more from those capital N you come up with capital X final successes and then there is an expectation issued there.

But we will come to that later. So, the first m is a deterministic non-random number but the subsequent N and X they are random variables. So, the question is what is the distribution of N? What is the distribution of X? And how much do you expect to earn and what if you know p 1 is greater than p 2. So, the two success probabilities are different and. So, in that case does it matter who goes first. So, the first would be to understand that the random variable n the first set of successes from small n trials is actually a binomial random variable.

And it has its parameter capital its lowercase m and p 1. Now given that this capital N has attained a certain value a particular value a little n the random variable x is actually a binomial random variable as well. So, that is an important point to note that conditioned on capital N capital X is binomial. So, let us write that out. So, that's what you see on the screen the conditional PMF of X given little m.

Now which means; that we should be able to reduce the unconditional PMF of x by applying the theorem of total probability uh. So, that is what you see in the next block that the PMF of x is the sum as comes from the total probability theorem and you just plug in the known PMF of n into that except we just have to be cautious that n can never be less than x. So, I have put an indicator function there but you just have to keep in mind that anything less than any x less than n the terms in the sum those will be zero. So, let us move on I will go to the next slide and continue with the solution.

(Refer Slide Time: 04:29)



After doing some substitution which you can work through it is going to be a few lines we can arrive at the PMF of x in terms of a sum that we can further simplify and arrive at an expression that is going to be very familiar. And if you want to just work through this all the notations and etc and then let me go to the next line and that looks very familiar. So, what it looks like is nothing but a binomial PMF a binomial PMF whose one parameter is m and the other success probabilities p 1, p 2.

So, now this random variable x its unconditional PMF is also binomial just like n and its parameter is also m like the first case but its success probability is now multiplied by the two. So, it's not p 1 only is p 1 times p 2. So, now we have done this sort of problems already a few times. So, the earning is the fixed number the non-random number. So, it is minus m times ones 1 rupee per dot and the random earning.

So, whatever the value of x is that is what gives me the income? So, 10 times expectation of x and the earning is 10 p 1 p 2 - 1 whole times m and then you can work out what those values will be once you plug in p 1 p 2 and m. The final question is that does it matter who goes first it does not because the solution is symmetric in p 1 and p 2.