Structural Reliability

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Lecture –35
Common Probability Distributions (Part - 06)

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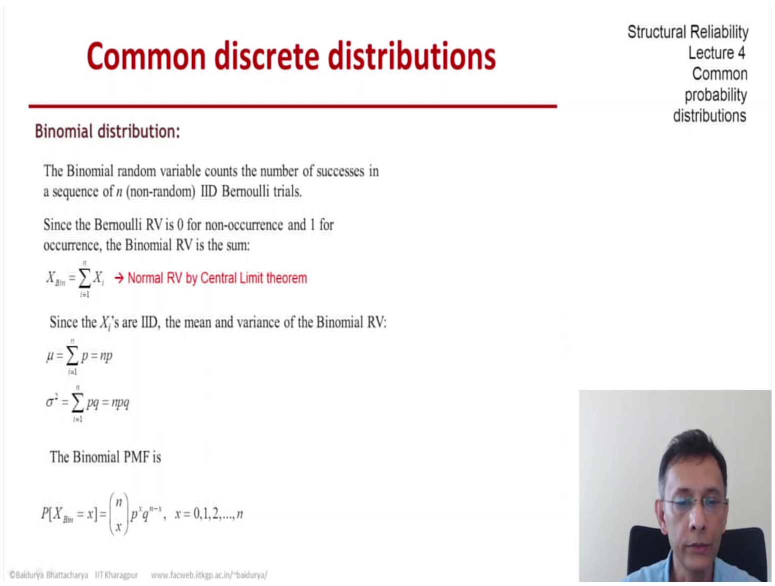

We next look at the binomial and the negative binomial distribution. In a sequence of independent and identical Bernoulli trials if I ask the question that what is the number of successes in a fixed number of n trials? Then the answer is the binomial random variable and I can express the binomial random variable as the sum of the n IID Bernoulli trials. Now this actually lets me derive the mean and the variance of the binomial random variable nicely since the expectation of the sum is the sum of the expectations.

So, the mean of the binomial is n times p and the variance of the binomial is the sum of the variances because the Bernoulli trials are all mutually independent. I can also derive the PMF of the binomial from the basic definition if I have a particular sequence of those n trials and X of them are successes. So, there will be n - X failures. So, the probability of that sequence would be p to the power of X times q to the power of n – x.

But there is not just one way of choosing that particular number of successes there are other sequences which could give the same number. So, there are n choose X ways of getting that sequence. So, my binomial PMF is what you see on this screen. Now we know that the binomial the limiting form of the binomial distribution is the normal distribution. There are ways of proving that but if you just look at this basic definition of the binomial as the sum of n independent Bernoulli trials.

Then you can invoke the central limit theorem and for a reasonably large value of n the X binomial must approach the normal distribution by central limit theorem.

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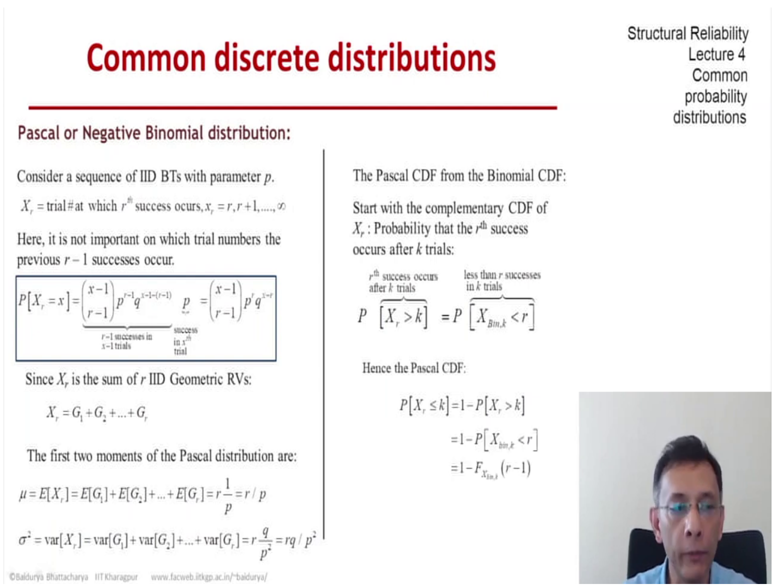

Now let us move on to the negative binomial distribution we again have the sequence of IID Bernoulli trials with parameter p but the question that we ask now is the number of trials is not fixed but I want to generalize the question I asked in the geometric case is what is the trial number at which success number r would occur. So, r is not necessarily one if it's one then I have the geometric but r could be a higher number.

So, the random variables are the negative binomial random variable and let us see how we can derive its PMF? The thing to remember is that it's not important on which trial numbers the previous r - 1 success occur it is the success number r that we are most interested in. So, the probability that X r is equal to that particular X is that we choose r - 1 successes in the X - 1 trials and the trial number X then has to be a success if X r has to equal little x.

So, if you do the algebra then the PMF of X r is what you see on the screen. There is another way of looking at this also is the negative binomial random variable is the sum of our independent and identical geometric random variables. And by the same logic this would also make the negative binomial distribution approach the normal distribution for reasonably large values of r. Now again by looking at this sum we can easily get the first two moments the mean and the variance of the Pascal distribution which is r divided by p and the variances likewise you can do the sum it is r q over p squared.

The Pascal CDF can be derived from the binomial CDF if you just express the if you just start with the complementary CDF of f X r that the probability that the success number r occurs after k trials is that there are less than our successes in k trials those are equivalent statements. And if we do the algebra we arrive at the Pascal CDF in terms of the complementary binomial CDF evaluated at r - 1.