Structural Reliability
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Lecture –34
Common Probability Distributions (Part - 05)

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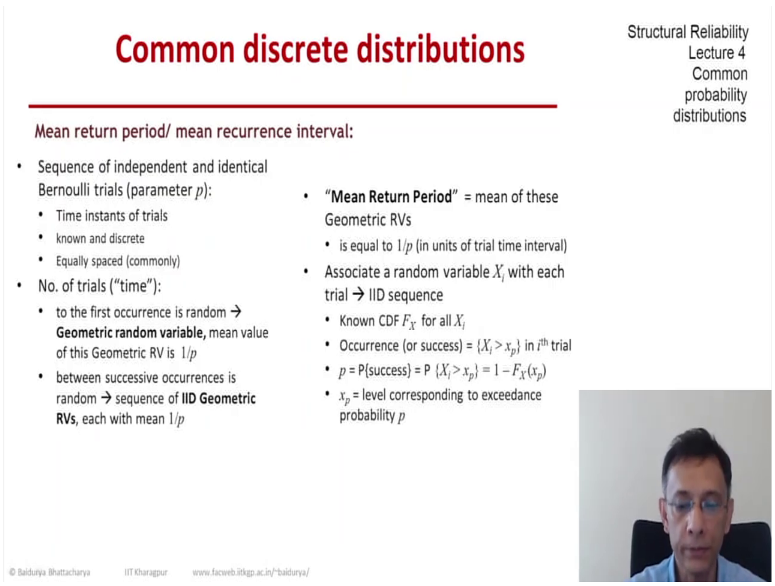

The concept of mean return period or mean recurrence interval is closely linked to the geometric distribution. So, let us recall we have a sequence of independent and identical Bernoulli trials with parameter p which means the success probability or occurrence probability is p. And the time instants of trials are known and they are discrete and commonly we think of them as equally spaced. Now the question that we asked when we defined the geometric random variable is what is the time or more correctly the number of trials to the first occurrence.

And that is a geometric random variable and its mean is 1 over t. Now I could also be interested if I let this sequence of Bernoulli trials run that what would be the time between successive occurrences. These actually are also geometric random variables these times between successive occurrences and they are IID geometric. So, each of these is a geometric random variable with the same mean one over p and they are placed one after the other.

So, the mean of these geometric random variables is the mean return period and since these are

IID the mean value is 1 over p. So, that is what we known as the mean return period or the mean recurrence interval. Now you will see and it gives us more flexibility and power actually to associate a random variable with each trial. So, if we associate random variable x i with trial number i and these random variables now constitute the IID sequence because these random variables are independently independent and they are identically distributed.

So, with the common CDF f(x). So, this occurrence probability of the success probability now is defined in terms of this associated random variable which if it exceeds a certain threshold x p. So, that gives the probability of p of success. So, the random variable x i exceeding the threshold of x sub p is the success probability in the trial number i and that is what we call p. So, that is 1 minus CDF of x evaluated at x_p. And this x_p now lets us define what the exceedance probability p is and in turn the mean return period which is 1 over p.

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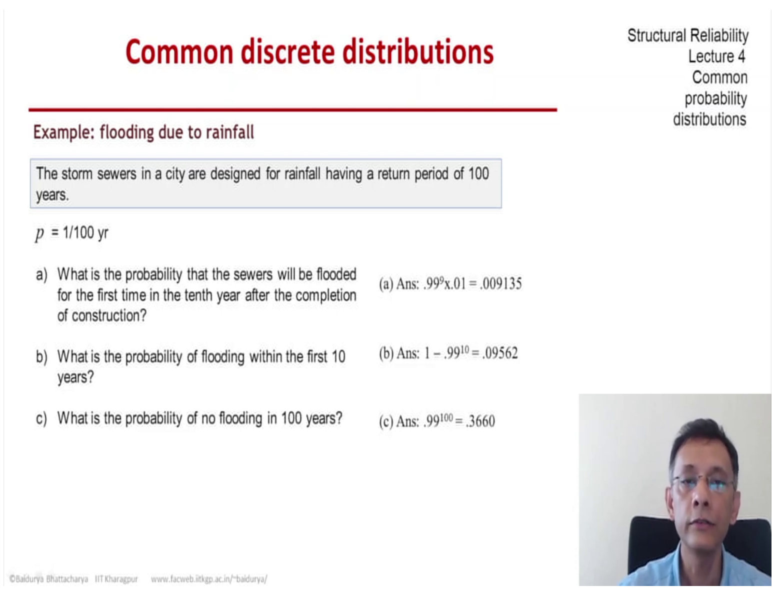

Let us take a look at one example. Let us say the sewers in our city have been newly constructed and designed for a rainfall having written period of 100 years. So, the first interpretation of this would be this annual exceedance probability is that 1 over 100 years. So, p is 0.01 per year. So, that is very important. So, we will ask three questions the first one is the probability that the first flooding will occur on year number 10.

So, it is a simple question involving the geometric distribution and if you want to work through the problem please pause the video otherwise let me present the answer and that is 0.99 to the power of 9 times 0.01, so 0.009135. The next question is that there will be flooding within the first 10 years. So, it is not asking whether the flooding will occur in year number 10 but any time between 1 and 10.

So, that would be easy if we look at the complementary event and the answer is about 9.56% we can ask one more question but let us look a little longer duration and the probability there will be no floods in 100 years and that would simply be 0.99 raised to the power of 100 and as you can guess that number approaches the classical limit of 1 over e.