

Structural Reliability
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Lecture –33
Common Probability Distributions (Part - 04)

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Common discrete distributions

The geometric distribution:

The Geometric random variable, G , represents the trial number of the first success in a sequence of IID Bernoulli trials $\{X_i\}$:

$$\{G = n\} = \{X_1 = 0, X_2 = 0, \dots, X_{n-1} = 0, X_n = 1\}$$

Probability law

Since the sequence $\{X_i\}$ are IID, the PMF of the geometric random variable is easily derived from its definition:


$$\begin{aligned} P(G = n) &= P\{X_1 = 0, X_2 = 0, \dots, X_{n-1} = 0, X_n = 1\} \\ &= P\{X_1 = 0\}P\{X_2 = 0\} \dots P\{X_{n-1} = 0\}P\{X_n = 1\} \\ &= q \cdot q \dots q \cdot p \\ &= q^{n-1} p \end{aligned}$$

The CDF of the Geometric RV can also be derived easily from its definition:

$$\begin{aligned} F_G(n) &= P\{G \leq n\} \\ &= 1 - P\{G > n\} \\ &= 1 - P\{G \geq n+1\} \\ &= 1 - P\{X_1 = 0\}P\{X_2 = 0\} \dots P\{X_n = 0\} \\ &= 1 - q^n \end{aligned}$$

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Let us start with the geometry distribution. So, G is the geometric random variable and if I write out the event that G is equal to n what I basically mean is the first success occurs at trial number n . So, which is saying the same thing as $X_1 = 0$ which means the first one trial is zero the second vertical trial is zero all the way up to $X_{n-1} = 0$ and X_n is equal to 1. So, this is the expansion of the statement that G is equal to n .

Now we can then just use the independence of these successive Bernoulli trials all the X 's and write out the probability of the joint event as the product and then use the fact that they are all identical. So, $P\{X_1 = 0\}$ and $X_2 = 0$ all the way up to $X_{n-1} = 0$ they all have the probability of q and the last one has a probability of p and I can multiply all of them and the geometric PMF is q to the power $n - 1$ times p .

I can also find the CDF from very simple considerations I could add these first n terms and come

up with the sum the first n PMF's to come up with the CDF but I can also use a simpler logic to say that the CDF of G at little n is the G is less than or equal to n which means the first success occurs at trial number n or before. So, which is one minus the probability that the first the first success occurs after trial number n . So, that is $1 - q$ to the power of n .