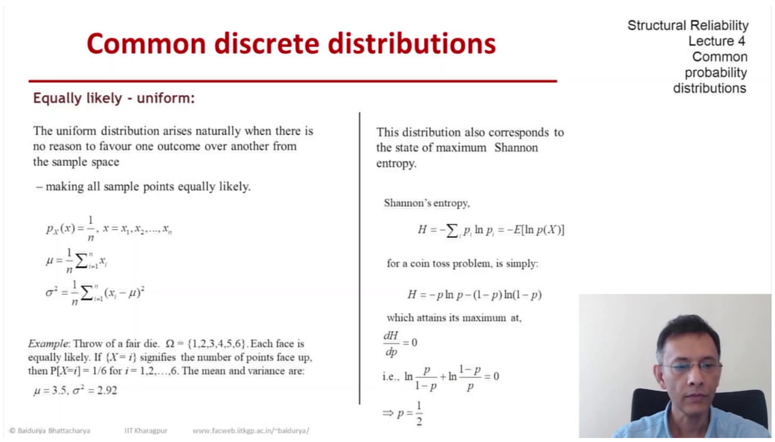
Structural Reliability

Prof. Baidurya Bhattacharya
Department of Civil Engineering
Indian Institute of Technology, Kharagpur

Lecture –32
Common Probability Distributions (Part - 03)

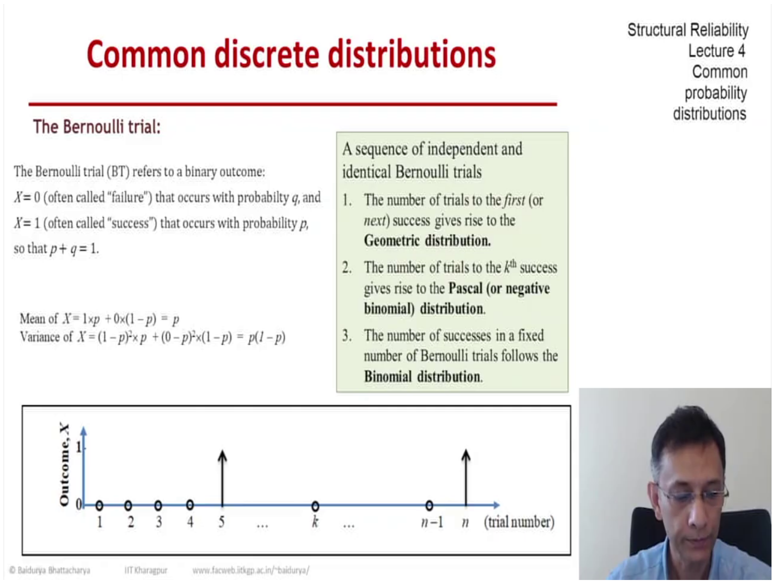
(Refer Slide Time: 00:28)



The discrete unity form arises naturally when all the sample points are equally likely. So, the pmf is just 1 over n for all the possible values and you can define the mean and variance as you see on the screen. We have discussed this example in the last lecture. So, if you have a fair idea then the six phases are equally likely and we can define the mean and variance as 3.5 and 2.92. Now this uniform distribution is also one that corresponds to the state of maximum Shannon's entropy which is defined as a sum of p log p.

And we can just walk through a simple example involving uh a coin toss. So, if the probability of head is p and the probability of tail 1 - p which value of p is going to make the entropy the maximum and we could just find it using basic calculus and the answer is half. So, it is those state the probative head being half and tail being half which maximizes the uncertainty.

(Refer Slide Time: 02:08)



The next we will look at would be the Bernoulli trial. The Bernoulli trial in itself is not as important as what a sequence of independent and identical Bernoulli trials can give us and that is three of the most important distributions that will encounter the geometric uh the binomial and the negative binomial or Pascal. So, let us look at the Bernoulli trial first it's it it has two outcomes two possible outcomes binary random variable.

It does not have to be but we most commonly assign the values zero and one to these two possible outcomes and we call zero as the failure and one as the success although success does not have to mean something good it could just be that something that occurs that can cause problems like the occurrence of floods or occurrence of earthquakes can be defined as the state of one in a Bernoulli trial.

So, one has a probability of p and 0 has the probability of 1 - p or q and you can define the mean and variance you can derive it simply by expanding the definition. So, the mean of the Bernoulli trial is p and the variance is p q now I mentioned that what is most important is a sequence of such Bernoulli trials when they are independent and identically distributed. So, what do we mean by that? Independent means that the different trials if you look at them in sequence they are mutually independent.

And identical distributed means that the success probability p remains constant from trial to trial.

So, what you see on the screen is just one such sequence the outcomes are either 0 or 1 on the y-axis and on the x-axis is the trial number. So, in this particular case you see that the first trial is a failure the second trial is a failure and until the fifth trial which is the first success and it goes on until we have success again in trial number n.

So, the three random variables that come out of this consideration is the first one is the answer to the question that when at which trial number is my first success going to occur. I can also ask a related question that what is the time between occurrences or successes or when is the next success going to occur after how many trials. So, this gives rise to the geometric distribution likewise i might be interested not in the first success.

But I might be interested in success number k and the question is that at which trial it occurs. So, that gives me the Pascal or the negative binomial distribution and if I have a fixed number of trials out of them how many are a success that gives rise to the binomial distribution.