Structural Reliability
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Lecture –30
Common Probability Distributions (Part - 01)

In this lecture we are going to talk about some of the discrete distributions that appear commonly in structural reliability theory. And after that we are going to take up the continuous distributions that appear commonly in this subject.

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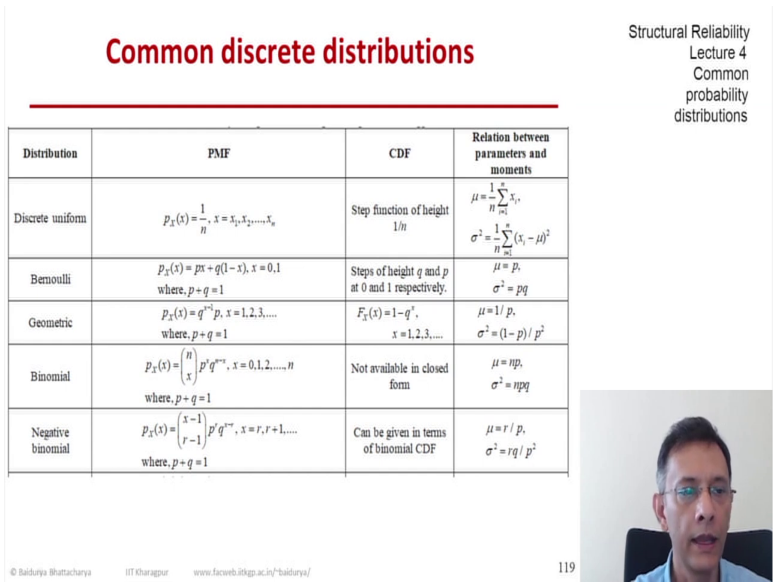

Distribution	PMF	CDF	Relation between parameters and moments
Discrete uniform	$p_X(x) = \frac{1}{n},\ x = x_1, x_2, \ldots, x_n$	Step function of height $1/n$	$\mu = \frac{1}{n}\sum_{i=1}^{n} x_i,$ $\sigma^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \mu)^2$
Bernoulli	$p_X(x) = px + q(1-x),\ x = 0,1$ where, $p + q = 1$	Steps of height q and p at 0 and 1 respectively.	$\mu = p,$ $\sigma^2 = pq$
Geometric	$p_X(x) = q^{x-1}p,\ x = 1,2,3,\ldots.$ where, $p + q = 1$	$F_X(x) = 1 - q^x,$ $x = 1,2,3,\ldots.$	$\mu = 1/p,$ $\sigma^2 = (1-p)/p^2$
Binomial	$p_X(x) = \binom{n}{x} p^x q^{n-x},\ x = 0,1,2,\ldots,n$ where, $p + q = 1$	Not available in closed form	$\mu = np,$ $\sigma^2 = npq$
Negative binomial	$p_X(x) = \binom{x-1}{r-1} p^r q^{x-r},\ x = r, r+1, \ldots.$ where, $p + q = 1$	Can be given in terms of binomial CDF	$\mu = r/p,$ $\sigma^2 = rq/p^2$

I have a list here and it is also going to be there in your course material. So, on this list we have first the discrete uniform and then the four which come next arise from the Bernoulli trial or a series of Bernoulli trials.

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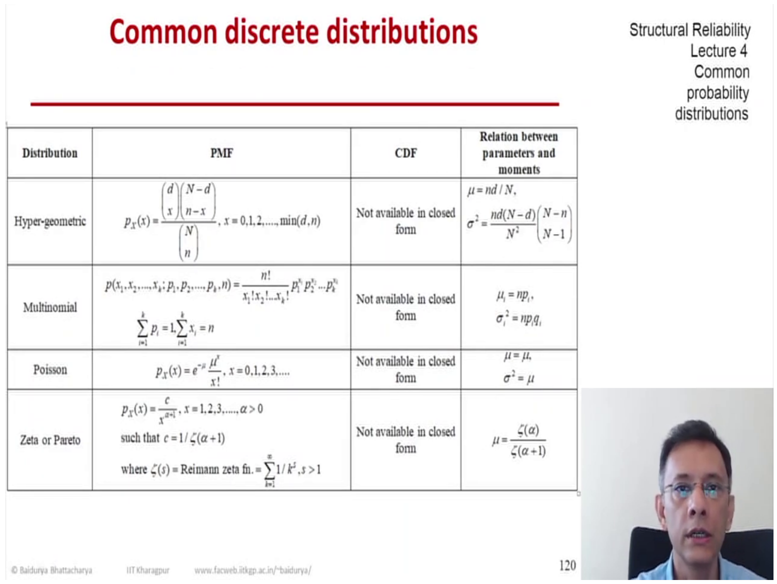

Distribution	PMF	CDF	Relation between parameters and moments
Hyper-geometric	$p_X(x) = \frac{\binom{d}{x}\binom{N-d}{n-x}}{\binom{N}{n}}, x = 0,1,2,\ldots,\min(d,n)$	Not available in closed form	$\mu = nd/N,$ $\sigma^2 = \frac{nd(N-d)}{N^2}\left(\frac{N-n}{N-1}\right)$
Multinomial	$p(x_1,x_2,\ldots,x_k;p_1,p_2,\ldots,p_k,n) = \frac{n!}{x_1!x_2!\ldots x_k!}p_1^{x_1}p_2^{x_2}\ldots p_k^{x_k}$ $\sum_{i=1}^{k} p_i = 1, \sum_{i=1}^{k} x_i = n$	Not available in closed form	$\mu_i = np_i,$ $\sigma_i^2 = np_iq_i$
Poisson	$p_X(x) = e^{-\mu}\frac{\mu^x}{x!}, x = 0,1,2,3,\ldots$	Not available in closed form	$\mu = \mu,$ $\sigma^2 = \mu$
Zeta or Pareto	$p_X(x) = \frac{c}{x^{\alpha+1}}, x = 1,2,3,\ldots, \alpha > 0$ such that $c = 1/\zeta(\alpha+1)$ where $\zeta(s) =$ Reimann zeta fn. $= \sum_{k=1}^{\infty} 1/k^s, s > 1$	Not available in closed form	$\mu = \frac{\zeta(\alpha)}{\zeta(\alpha+1)}$

The next we two that we will focus on would be the hyper geometric and the Poisson distribution. So, let us start with the discrete uniform.