

**Structural Reliability**  
**Prof. Baidurya Bhattacharya**  
**Department of Civil Engineering**  
**Indian Institute of Technology, Kharagpur**

**Lecture –29**  
**Review of Random Variables (Part - 12)**

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## Review of random variables

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**Moment generating function:**

$$G_X(s) = E[e^{sx}] = \int_{-\infty}^{\infty} e^{sx} f_X(x) dx$$

It exists if the integral is finite for all  $s$  in some interval  $I$  that contains 0 in its interior.  
 If it exists, the MGF uniquely determines the distribution of  $X$ .  
 In comparison, the CF of a RV *always* exists.

MGF is infinitely differentiable and the  $n^{\text{th}}$  derivative,

$$\frac{d^n}{ds^n} G_X(s) = \int_{-\infty}^{\infty} x^n e^{sx} f_X(x) dx$$

evaluated at  $s = 0$ , gives the  $n^{\text{th}}$  raw moment of  $X$ .

$$\left. \frac{d^n}{ds^n} G_X(s) \right|_{s=0} = E[X^n]$$

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If instead we perform successive derivatives on  $\ln G_X(s)$  and evaluate them at  $s = 0$ , we get the central moments of  $X$ :


$$\left. \frac{d}{ds} \ln G_X(s) \right|_{s=0} = \mu_1$$

$$\left. \frac{d^2}{ds^2} \ln G_X(s) \right|_{s=0} = \sigma_2^2$$

$$\left. \frac{d^3}{ds^3} \ln G_X(s) \right|_{s=0} = E[(X - \mu_1)^3]$$

etc.

where the identity  $G_X(0) = 1$  has been used



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The moment generating function is the expectation as you see on the screen the expectation of exponential of  $s x$ ,  $s$  is a real number and  $x$  is the random variable. Now if it exists then we can retrieve moments of all orders of the random variable from the generating function and in particular if you take the  $n$ th derivative of the homogeneity function and evaluate it at 0 for  $s$  you get the moment of order  $n$ .

And if you start with the log of the moment generating function you would get the central moments. Now there are other functions other expectations like the cumulative generating function which is the log of the characteristic function. The probability generating function for integer value random variables etc but we will not discuss them.

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# Review of random variables

Fundamental properties of distributions:

### Markov's inequality

If  $f_x(x) = 0$  for  $x < 0$ ,

$$P[X \geq \epsilon\mu] \leq \frac{1}{\epsilon}, \quad \epsilon > 0$$

### Bienyame Inequality

$$P[|X - \mu| \geq \epsilon^n] \leq \frac{E[|X - \mu|^n]}{\epsilon^n}, \quad \epsilon > 0$$

$n=2$  reduces to Chebyshev's inequality.

### Chebyshev's inequality

$$P[|X - \mu| \geq \epsilon\sigma] \leq \frac{1}{\epsilon^2}, \quad \epsilon > 0$$

### Lyapunov Inequality

Let  $\beta_k = E[|X|^k] < \infty$  where  $k \geq 1$  is any integer, then

$$\beta_{k+1}^{1/(k+1)} \leq \beta_k^{1/k}$$

equivalently,  $\beta_n^m \geq \beta_m^n$  for  $n \geq m$



Let us move on to some non-parametric properties very powerful properties of random variables that involve some expectation of the other. The first one of them is the mark of inequality which basically tells you that as you move away from the mean you are going to find less and less probability content. The Chebyshev's inequality says the same thing which is generalized by the Bienyame inequality which reduces to Chebyshev's for  $n$  equals 2 and finally Lyapunov of inequality which gives a hierarchical order involving the expectations.