

Structural Reliability
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Lecture –28
Review of Random Variables (Part - 11)

Let us end this review of random variables with discussing a few advanced topics involving expectations. At the beginning of this lecture I mentioned that the probability law of random variables can be described variously with the help of distribution functions probability density or mass functions as well as characteristic functions and moment generating functions.

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Review of random variables

Characteristic function:

The PDF/PMF and the Characteristic Function form a Fourier Transform (FT) pair.

$$\psi_X(\theta) = E[\exp(i\theta X)] = \int_{-\infty}^{\infty} e^{i\theta x} f_X(x) dx$$

$$f_X(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \psi_X(\theta) e^{-i\theta x} d\theta$$

If f_X is discontinuous at some x_1 , this equals $\frac{1}{2}[f_X(x_1+) + f_X(x_1-)]$.

The requirement is that f_X is absolutely integrable, i.e., $\int_{-\infty}^{\infty} |f_X(x)| dx < \infty$, which is no problem since f_X is a PDF to begin with.

The characteristic function of a RV always exists and uniquely determines its probability distribution.

Properties of CF:


- $\psi_X(0) = 1$
- $|\psi_X(\theta)| \leq 1$
- $\psi_X(\theta)$ is uniformly continuous
- $\psi_X(\theta)$ is real $\Leftrightarrow \psi_X(\theta) = \psi_X(\theta)$ $\Leftrightarrow f_X(x) = f_X(-x)$ (symmetric density)
- If two RVs X, Y have $\psi_X(\theta) = \psi_Y(\theta) \Leftrightarrow X = Y$ (in distribution)

Two RVs X, Y are equal in distribution if and only if they share the same probability measure:

$$P[X \in A] = P[Y \in A] \quad \forall A \in \mathbb{R}$$

Further reading: A Gut, Probability a Graduate Course, Springer

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So, the characteristic function is a certain expectation of the random variable x and actually it is the Fourier transform of the density and the density of the inverse transform of the characteristic function and if the random variable is discrete specifically integers then you can also describe the Fourier transform pair and since the density function is absolutely integrable. So, that is that poses no problem for some of the issues involved with the existence of the Fourier transform.

Now one of the utilities of characteristic function comes from the fact that it always exists. And this is relevant because our next topic the moment generating function may not exist in all cases the notable example always being the Cauchy distribution the caution distribution does not have

any verbal none of the moments is defined. Here are some of the properties of the characteristic function and if you need to read further in addition to the books I mentioned at the beginning of this lecture this one that I have listed here is also excellent.