Structural Reliability
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Lecture –26
Review of Random Variables (Part - 09)

Since we have been discussing probability laws and expectations and we have discussed conditional probabilities and conditioning events it would not be a bad idea at this point just to introduce conditional densities, conditional distributions and conditional expectations.

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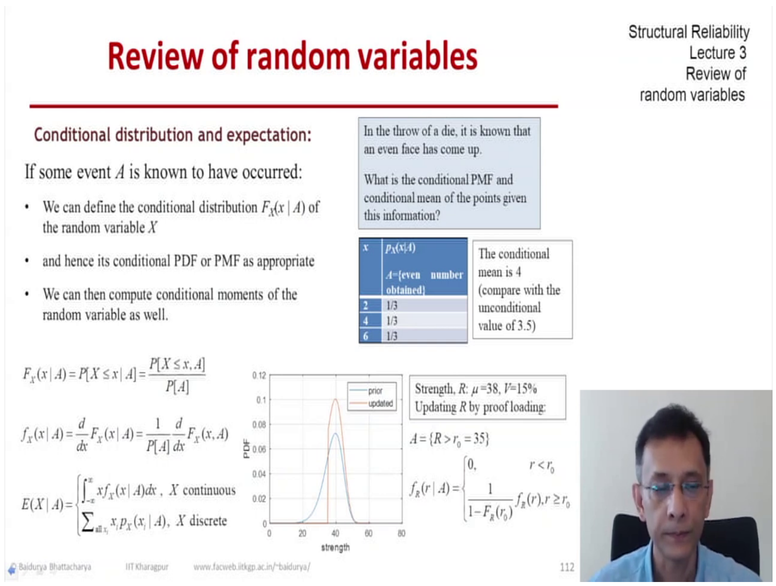

Although the full utility and impact would be clear when we discuss joint distributions. So, let us say that some event A is known to have occurred that some information becomes available and we may be interested to know how it affects a random variable x. So, when we do such conditioning some outcomes of the random variable become more likely and others less. So, unless of course the; random variable is independent of the given input.

So, in general so, we can define the conditional distribution of the random variable given the event A and if we can do that we can define the conditional density function or mass function as relevant we can also compute the conditional expectations and other moments of the random variable as well. So, here is the expression for that. So, now we have F of x given A the CDF of x given a and defined simply as the probabilities in the numerator we have x less than little x and A and the denominator we have P of A the probability of the conditional moment itself.

And then we can if it is a continuous random variable we can take the derivative of that to get the conditional density function and we can integrate that with respect to X to get the conditional mean or if it is a discrete random variable we can sum to get the conditional mean of the random variable given A. So, let us look at a couple of examples. So, suppose in the throw of a die which we have discussed a few slides back it is known that an even face has come up.

So, our conditioning event is the face is even and so we are looking now at a restricted part of the sample space. So, what is the conditional PMF and what is the conditional mean given this information. So, clearly we now have only 2, 4 or 6 as the possibilities and the probabilities of zero, I am sorry probability of 1, 3 and 5 are zero. So, we need to make sure that in this conditional mass function they all add up to 1 as they always should.

And if you look at the conditional mean it is simply 4 which is different from the value of 3.5 which we found in the earlier example where it was unconditional. We can do we can look at one more example involving continuous random variables. For example let us say we have the strength of a member with you know whose mean is 38 units and COV the coefficient variation is 15 and suppose we update the strength of this member by proof loading.

So, we perform a proof loading of a known value and find that the member survives. So, what is the updated density function of the strength given this information? So, our conditioning event A is that the random variable r is greater than the proof load which in this example let it be 35 units. So, now our density function clearly has two regions one is zero for all values of less than the proof load because we know that it survived the proof load.

And for values greater than or equal to the proof load it is the original density function but divided by the exceedance CDF. So, the complement of the CDF and if you look at the diagrams the plots of the density function you can see that the blue line is the original the prior density of the strength whose mean was 38 and. Now that I know this cannot be less than 35 the orange line is the updated density.

So, you can see that no density is there left of 35 and the area under the orange curve is still 1 as is the area under the blue curve. So, it is now the probability is more concentrated to values on the right of 35.