

**Structural Reliability**  
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**Lecture –239**  
**Target Reliabilities (Part - 02)**

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Lecture 34  
Target reliabilities


### Recap: What is engineering

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- An engineering project
  - Functional objectives
  - Safety constraints
  - Resource constraints
    - Time
    - Money
    - Knowledge
  - Design
    - Objectives are met
    - Constraints are satisfied
    - Tradeoffs/ compensations made
  - Execution

```
graph TD; Input[Input] --> MM[Mathematical model]; MM --> Output[Output]; Output --> Results{Results OK?}; Results -- N --> Modify[Modify]; Modify --> MM; Results -- Y --> Next[Move to next stage  
(prototyping/  
construction/  
production)];
```

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Let us go back to some ideas that we encountered early on in this course in fact in lecture one itself and let us build up on some of those ideas. An engineering project we discussed is always meant to serve or satisfy some performance objectives and while serving those objectives it should not be too unsafe. So, there are safety constraints and there are other constraints like there is limited time there is always limited funds and there is always limited knowledge.

Knowledge about what the mechanics or the physics of the system is the loads and how things might change in the future. So, with all these constraints and the need to satisfy the objectives the design process is an iterative process obviously and it continues until the objectives are met the constraints are satisfied. And invariably trade-offs and compensations are made until it is ready and the next stage would be depending on the system in question go for the prototype go for construction go for production etcetera.

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## Objectives and constraints in design

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An engineering system is intended to

- (i) satisfy its performance requirements, while being
- (ii) economical and
- (iii) safe.

The emphasis can be different depending on the stakeholder.  
→ Multi-objective optimization!

**Approach 1:** minimize the total cost:  

$$\min C_T(\underline{x})$$
subject to  $\theta_i(\underline{x}) \geq \theta_i^*$   $i = 1, \dots, m$   

$$M_i(\underline{x}) \geq M_i^*$$
  $i = 1, \dots, n$

$C_T$  = total cost  
 $B$  = total benefit  
 $\theta$  = set of performance measures that must be met  
 $M$  = set of safety margins  
 $\underline{x}$  = decision variables  
 $*$  = minimum acceptable value

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**Approach 2:** maximize performance:  

$$\max \theta(\underline{x})$$
subject to  $C_T(\underline{x}) \leq C_T^*$   

$$M_i(\underline{x}) \geq M_i^*$$
  $i = 1, \dots, n$

**Approach 3:** maximize net benefit:  


$$\max B(\underline{x}) - C_T(\underline{x})$$
subject to  $\theta_i(\underline{x}) \geq \theta_i^*$   $i = 1, \dots, m$   

$$M_i(\underline{x}) \geq M_i^*$$
  $i = 1, \dots, n$

**Approach 4:** maximize benefit cost ratio:  

$$\max B(\underline{x}) / C_T(\underline{x})$$
subject to  $\theta_i(\underline{x}) \geq \theta_i^*$   $i = 1, \dots, m$   

$$M_i(\underline{x}) \geq M_i^*$$
  $i = 1, \dots, n$



So, we can then restate that these ideas that an engineering system is intended to satisfy its performance requirements while being economical and safe. So, these are very general statements that are always true. Now what would be considered economical what would be considered safe what are the performance objectives and how to prioritize them that obviously would depend on the on the stakeholder.

And different stakeholders would view the same problem differently because their priorities could be different. And in any case this presents a multi-objective optimization problem and if we just focus on one single objective simplify the problem one way of stating the optimization problem would then be that minimize the total cost subject to satisfying all the constraints. So, minimize C T which is a function of decision variables X and subject to that all the performance measures are at least as much as the minimum acceptable value and the safety margin M is also greater than the minimum acceptable value.

So, we have put these constraints in the in a one-sided format but it is quite easy to give it two-sided or an equality constraint as well. So, theta are the performance measures M are the safety margins and the stars the asterixes indicate the minimum except value when you have a one-sided constraint. But that is not the only way of describing such an optimization problem or an engineering solution we could have approached two in which instead of minimizing the total cost we can maximize performance some performance theta and now subjected to the cost being less

than or equal to the budget.

And obviously safety constrained the safety margin at least as much as the minimum acceptable value. Now maximizing performance subject to a budget as opposed to minimizing cost subject to minimum acceptable performance this approach to could also be an acceptable format may not be for publicly funded activities but for some one's personal use this could be very much of a viable objective an optimization formulation.

One could bring in benefits also in consideration. So, B is the total benefit and one could maximize the net benefit subject to the same constraints as we had in approach 1 one could also maximize the benefit cost ratio and subject to the same type of constraints. So, approach 3 and 4 they bring in the benefit in the consideration as well. Now one could also maximize safety you might be wondering the point to note there is that it is not very common that safety is an objective which can be which needs to be maximized for one thing that solution may not be safe enough.

So, there could always be some minimum standard of safety that comes from other consideration than from an optimization approach and for another typically economy and performance sell more than safety but when safety does sell obviously it is a performance requirement. So, one could have certain situations where maximizing safety was also a viable objective subject to constraints.

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The emphasis can be different depending on the stakeholder.

→ Multi-objective optimization!

- Need to properly define and account for costs and benefits.
- Whose cost is it?
- What costs to include?
  - Construction? Operation? Maintenance?
  - Demolition and Failure costs?
  - Environmental and ecological costs?
- Whose benefits are to be considered?
  - Public's, or shareholders', or the owner's?
- What about uncertainties? ←

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Now then some questions naturally come. So, we need to define all these costs and benefits which actually are either the objective or sometimes the constraint also. So, the question would be whose cost is it that would give rise to possible different answers and possibly different solutions. So, whose cost and what costs are to be included if you are talking about a structure then construction cost operation cost maintenance cost do we include demolition and failure costs also. Do we include costs to the environment and the ecology.

And then if we bring in benefits whose benefits are being considered some is it the owner's benefit is it the public's benefit or is it someone else's. And then obviously the big elephant in the room and which is something we have been grappling with throughout this course is what about uncertainties. So that is obviously the central consideration in many of the things we have done in this course.

And so let us then go back to the other slide that has come back frequently during these lectures and that is the course in one slide.

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## Objectives and constraints in design

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**Approach 1 revisited:** minimize the total *expected* cost

$$\min E[C_s(\underline{x})]$$
$$\text{subject to } P[\theta_i(\underline{x}) \geq \theta_i^*] > p_i, \quad i = 1, \dots, m$$
$$P[M_i(\underline{x}) \geq M_i^*] > R_i^*, \quad i = 1, \dots, n$$

$\{R_i^*\}$  = set of "target" (minimum acceptable) reliabilities

- For system and components
- For life safety limit state
- For serviceability (functionality) limit states
  - reversible
  - irreversible
- How to set target reliabilities?
  - in life safety
  - in functionality



So, this is our system and we want to build it or assess it we know what it is for what is supposed to do we know its environment we know its service life we know its behaviour. So, we know the system properties that are relevant we know the inputs that need to be considered then obviously we need to know what response we are interested in. So, you see an example there the tip deflection and which means that the response in terms of the system properties and the inputs are well defined.

And then we should know what the system capacity is or what the acceptable performance limit is. So, that we can say we can decide very clearly whether the system is serving its purpose or not. So whether there is failure or not this could be given in terms of one-sided limit or two-sided limit and we have discussed this quite at length. Now there could be multiple performance requirements that we have also handled. So, we are able to do that.

Now the presence of uncertainty, so, the uncertainties are central to what we are doing here. So, they could be present in the inputs the properties and the input output model and because of these uncertainties we have non-zero probability of failure and we compute probability of failure once we know the performance function once we know the uncertainties and then the question is, is it acceptably low and can I standardize the process is basically the design code issues which we looked at last week.

So, the central question in today's discussion and in this week's discussion is what is acceptable. So, that is why we are discussing target reliabilities and how to set them. So, obviously the way we set up the optimization problem in the previous slide and then ask the question is what about uncertainties. So, let us see how the presence or acknowledgement of uncertainties would change the optimization problem definition.

So, if we revisit approach one in which we minimized the total cost. So, now it would be minimize the total expected cost and. So, instead of just cost we have expectation of the total cost and the failure probability is implicit there and if we have just a binary outcome failure and no failure. So, the failure cost is typically huge if there is failure but with a low probability and if there is no failure cost if there is no failure then obviously the failure cost is zero.

There would be demolition cost maybe and so, that would give me the expected total cost I could bring in many other aspects of cost maintenance cost operation cost and so on. So, over the life of the structure I would have the total random cost I could find its expectation and that is what I would need to minimize using the decision variables  $x$ . And then just like we had the performance objectives had to be met now they have to be made probabilistically.

So, the probability that the performance would be at least as much as the minimum acceptable value should have a lower probability. So, that is the restating of that constraint in the probabilistic format. And the second set of constraints are actually what we have been worrying about and discussing a lot is that the probability that the safety margin is satisfied is at least met with the probability of  $R^*$ .

So, that  $R^*$  obviously we have recognized are the set of minimum acceptable reliabilities the word target might sometimes be a misnomer because we are not going to go for that as the value that we want for our reliability but it could often be and sometimes more intuitively be the minimum acceptable value of reliability than what to shoot for. Now this our star this target reliability then obviously would depend on and would change for system to different systems or for different components in the same system.

So, one might need to set depending on how the safety margins are defined the target reliabilities for systems and components for different types of limit states because the consequences are different for life safety versus functionality which is often called serviceability and whether that functionality limit state is reversible or irreversible would give a different measure and value of the acceptable or target reliability.

Now the central question is that how to set targeted liabilities and since we have brought out these two limit states explicitly and it makes sense to do. So, that we need to set target liabilities for life safety ultimate collapse and for those class of limit states and for serviceability or functionality class of limit states. So, let us tackle them one by one next.