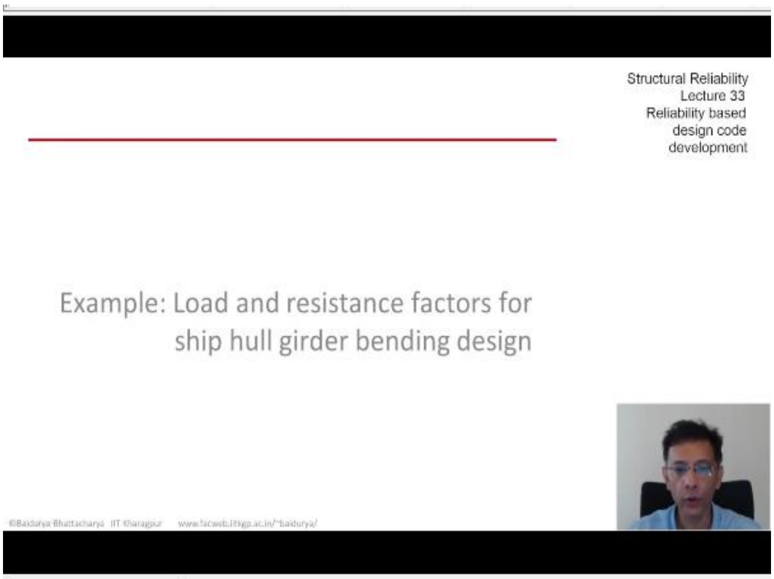
Structural Reliability
Prof. Baidurya Bhattacharya
Department of Civil Engineering
Indian Institute of Technology-Kharagpur

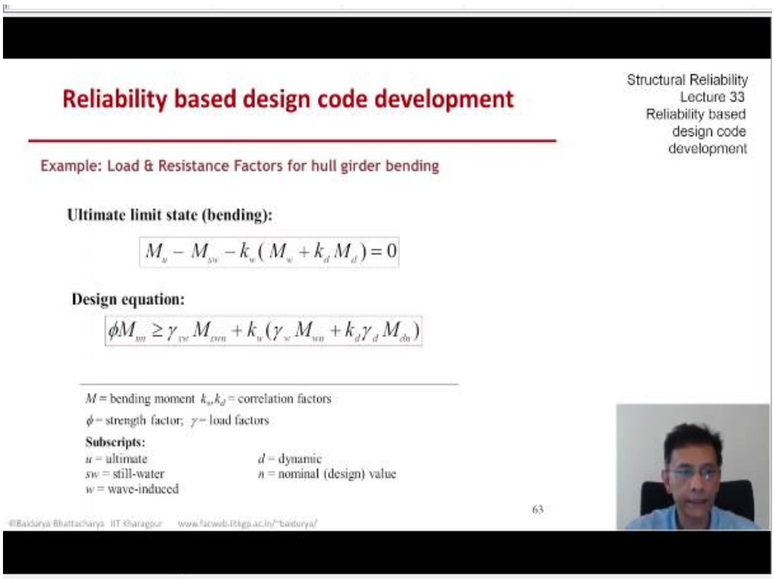
Lecture-237
Reliability Based Design Code Development (Part-04)

(Refer Slide Time: 00:26)



We now go through an example on deriving load and resistance factors for the design equation for ship hull girders in ultimate bending limit state.

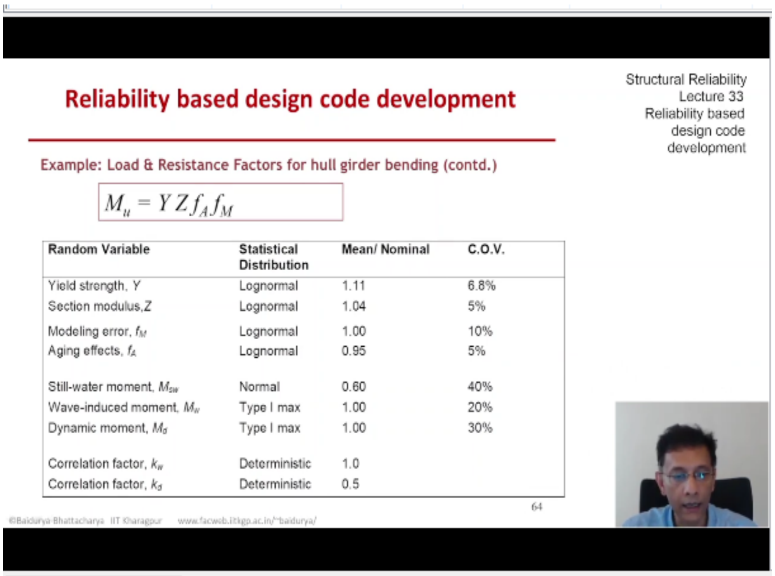
(Refer Slide Time: 00:43)



So, this is the limit set equation and it has been decided that in this load combination the still water, bending moment, the wave induced bending moment and the dynamic bending moments they are important and M u is the ultimate bending moment capacity. So, the design equation follows closely the limit state equation and we have phi times the nominal capacity must be greater than the factored loads. So, the still water bending moment M swn and then the wave induced and the dynamic bending moments.

Just to put everything together our subscripts are u for ultimate, sw for still-water, w for wave induced, d for dynamic and all the nominals have as we have been using n for the subscript. The phi is the strength factor, gamma are the load factors with the appropriate subscripts and there are 2 load correlation unload coincidence factors kw and kd that it has been decided based on the experience of analysts and designers that they should be there. So, now we define we take a little closer look at this ultimate moment capacity M u and let us see if we could split that into further detail.

(Refer Slide Time: 02:38)



So, an understanding of the mechanics tells us that we could define M u as the product of 4 random variables and they are the yield strength Y and the yield strength as we have been normalizing has a normal distribution with a bias of 1.11 and a COV of 6.8%. So, we do the same way we normalize, we use normalized statistics because soon we are going to normalize the limit state equation with the design equation.

The second factor in the moment capacity is the section modulus of the hull girder and that is also log normal with a bias of 1.04 and COV of 5%. Then there are 2 factors the modeling error factor f M and an aging effect factor f A, again each log normal and with bias factors 1 and 0.95 respectively and uncertainty of a rather moderate nature. So, what they do is they explicitly acknowledge the fact that this sort of simplification of the expressing the capacity in terms of Y times Z.

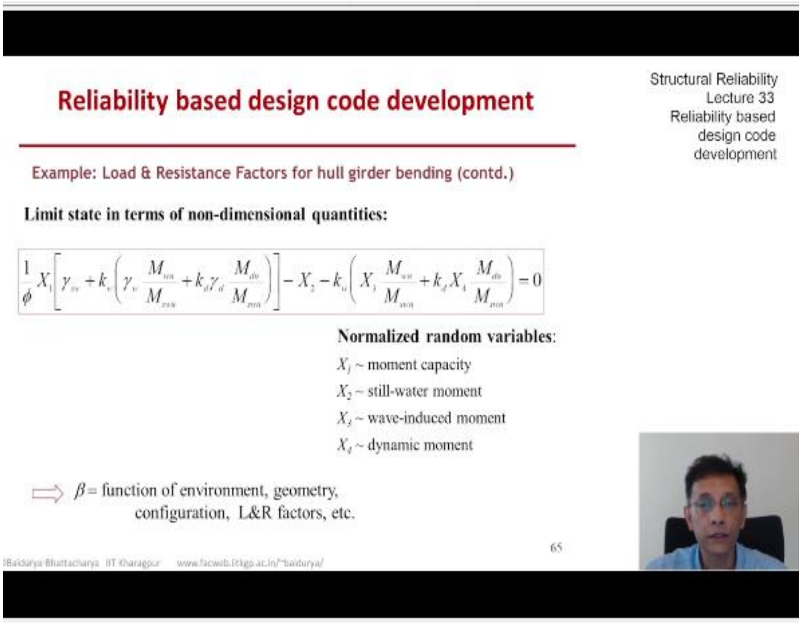
Yield strength time intersection modulus ignores many aspects it might ignore local geometric effect, it might ignore material hardening effect. So, all of that maybe through experiments, maybe through analysis, more sophisticated analysis it has been found that the modeling error is on the average, there is no error but there is some deviation of scatter around that and that is captured by the COV of 10%.

The aging effect likewise takes into account in-service corrosion and etcetera. So, for the duration that we are interested in the aging effect actually brings down the strength by a factor of an average 95% it brings down 2 not by but 295% with a certain uncertainty only 5%. So, and these are all lognormal and presumably they are all statistically independent. On the load side we have the still-water bending moment which kind of is like a dead load.

So, that has a normal distribution with rather high uncertainty of 40% and rather conservative estimates. So, the mean over nominal is only 0.6. So, the nominal substantially over estimates the quantity. The wave induced bending moment and the dynamic bending moment they do not have any bias in the sense the mean is equal to the nominal, each is gumbel distributed or type 1 distributed and they have COVs of 20, 30% respectable.

So, lesser than what we find for the still-water moment. Now the 2 other factors the load coincidence factors and load correlation factors they are both deterministic and they have the value of 1 and 0.5 respectively. So, in some sense they discount the effect of all these high loads acting together on the structure simultaneously.

(Refer Slide Time: 06:41)

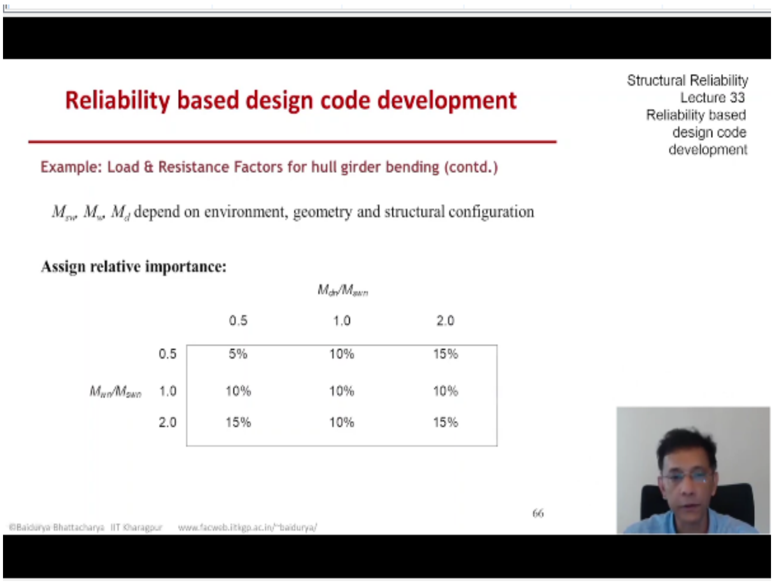


Now we are ready to normalize the limited equation so as we had done for the cable case, so because the capacity and demand quantities are separable, it is easy to normalize them and what you see are the non-dimensional X's random variables. So, X 1 is the capacity and X 2, X 3 and X 4 are the various loads and the load ratios which again are representative of various design situations are given with the still-water nominal moment in the denominator.

So, we have M wn over M swn M dn over M swn and M wn over M swn. So, there are 2 of them. Now we just again emphasize that the beta that would be obtained by analyzing this normalized limit state beta could be obtained by from the Pf the failure probability through the normal distribution function. We do not have to do a formal analysis we can do Monte Carlo or any other variance reduction techniques.

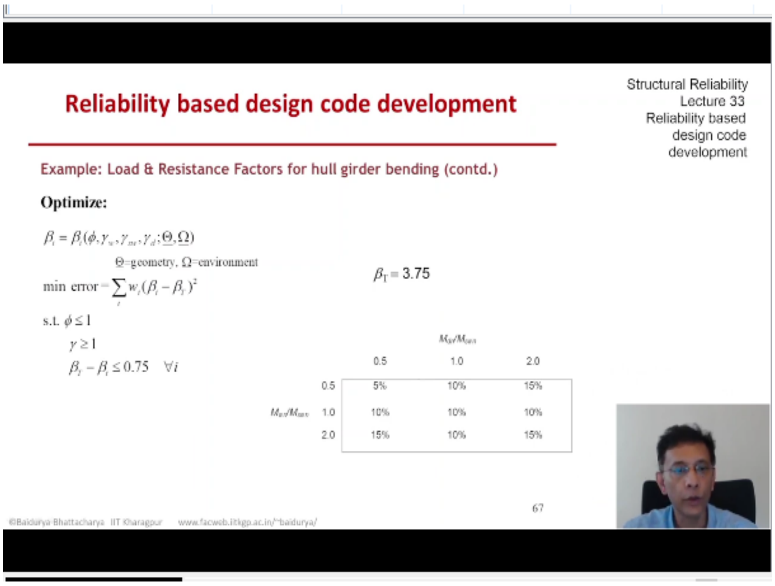
So, this beta when we obtain it would be a function of the environment meaning the statistics of the loads, the geometry, the configuration which means all these different nominal load ratios and obviously the loaded resistance factor. So, all the phi and the grammars are together they are going to give us the value of beta.

(Refer Slide Time: 08:35)



So, we discuss the weights or the relative importance of these various loading situations so this the still-water, the wave induced and the dynamic moments these are all the applied loads depend on the environment, the shift geometry and the structural configuration. So, just for illustrative purposes let us say there are 3 possible values of the ratio dynamic over still-water and 3 possible for the ratio of still-water of wave induced over still-water and we have this 3 by 3 table. If we add those 9 weights we should get a value of 1 and that is what we see here.

(Refer Slide Time: 09:30)

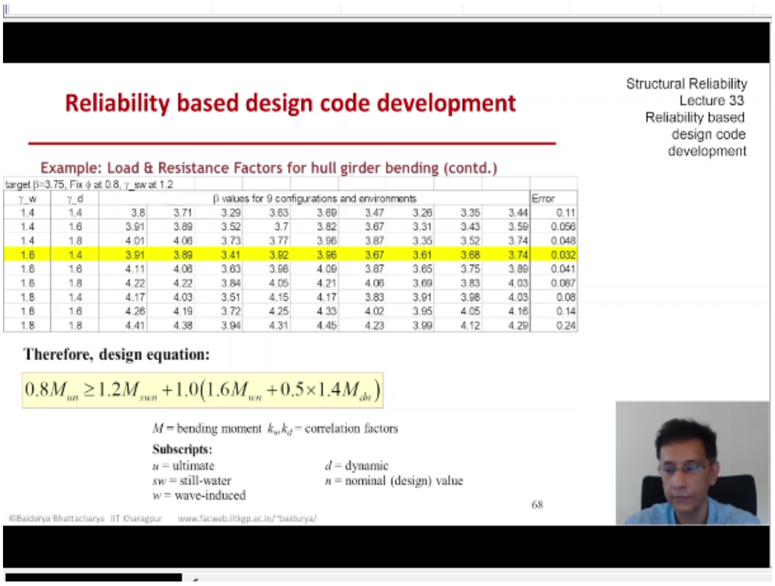


So, now we are ready to state the optimization problem and that is something we have seen before. So, beta for each of those cases of those 9 cases is a function of the load and resistance factors and all the statistics and the and load ratios that govern the design. We want to minimize the error which is beta minus beta target square for each of the i's multiplied by the weight and we sum all these weighted errors.

We would like to put some constraints. So, the phi has to be less than or equal to 1, the gammas we insist that they are all greater than 1 and we do not want any beta to be below 0.75 below the target and then let us bring in the weights, we know the weights and then we select a target reliability of 3.75. So, with this we can set up an opposition problem and which I m not going to go into the details now that has all been done.

We are able to do that, we are able to find betas for every possible combination of phi and gammas and the nominal load ratios. So, suppose we have done that and now let us present a solution scheme or in at least in a simplified manner.

(Refer Slide Time: 11:15)



So, what you see on your screen next is just a much smaller extract of a much larger exercise that should be undertaken. So, our target beta is 3.75, let us say we have fixed phi at 0.8, we just want to reduce the dimensionality of this optimization problem. So, that we can show it on one table in one slide and we have fixed the still-water load factor at 1.2. So, now we are left with just 2 of the other load factors gamma w and gamma d.

And for each row then we see the beta values for the 9 cases; the nine cases meaning the 9 load ratios that 3 by 3 table that we showed. So, those are the 9 beta values for each row, each row being defined by a pair of gamma w and gamma d and here we just run both of them from 1.4 to 1.8 again. In an actual optimization problem the whole decision variable space can be larger and there are obviously more decision variables than 2 but here they suggest for example purposes.

And we see on the very last column we have the weighted error, the error that we defined as sum of w times beta minus beta target squared for i going from 1 to n. So, that last column gives the error and we see from the gammas of 1.4 and 1.4 starting with 0.11, it goes to 1.8, 1.8 an error of 0.24, but somewhere in the middle we have the lowest error and that is 0.03 corresponding to a gamma w of 1.6 and a gamma d of 1.4.

So, if we are happy with this if our optimization problem, the objective function and the constraints they are what we like we have no problem with that. So, then this would be the solution that we should report and use in design. So, putting all of that together we have already fixed as I said phi at 0.8 and gamma sw at 1.2. Obviously, we do not need to do that in an actual exercise we would let them vary within the constraints.

But this is what the equation looks like and for again for your reference I have given all the definitions of the loads and the subscripts once again. So, this would be the process in which we could find a design equation involving partial safety factors that over a large range of design situations for a class of structures or structural elements would on an average satisfy a given target reliability.