Structural Reliability
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Lecture-235
Reliability Based Design Code Development (Part-02)

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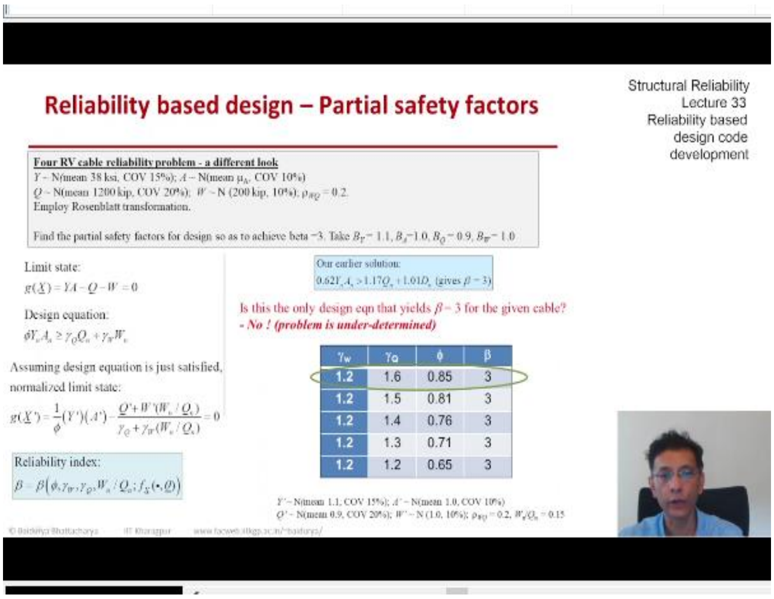

To recap we have our limit state equation on the left of your screen Y A - Q - W = 0. So, that is a 4 variable limit state problem, the corresponding design equation is phi Y n A n, so n being the nominal values, phi being the resistance factor or the material factor is greater than or equal to gamma Q Q n + gamma W W n. So, gamma Q and gamma W are the 2 load partial factors.

So, by assuming that the design equation is just satisfied we normalize the limit state with both sides of the design equation and we obtain a normalized or non-dimensional limit state equation in terms of the basic variables X prime. So, we could find P f as P g X prime less than 0, all the X primes are defined on the right bottom corner that you see, Y prime is normal with mean 1.10 with 15% so on.

So, the means are all the bias factors now and we also have fixed the nominal load ratio W n over Q n at 0.15. So, if we find P f and take the normal CDF inverse of that we would get the reliability index beta which you see on the bottom left of your screen is a function of the partial safety factors and the statistics of the problem and the nominal load ratio.

So, the question that we would like to answer now is our earlier solution from the previous lecture where we have the phi and the gamma values as you see on your screen on the right in the blue box is that the only possible design equation that yields beta equals 3. The answer is obviously not because we have 1 equation in 3 unknowns. So, the one equation is that beta equals 3 and the degrees of freedom that we have that we can play with are at least 3 phi gamma W and gamma Q if all the others are fixed.

So, let us just take one example, so let us fix the value of gamma W and let us change the value of gamma Q and then accordingly we would have to adjust phi in order to get a beta equals 3. So, if you look at the first row we take gamma W as 1.2, gamma Q is 1.6. So, that is a standard dead load factor and live load factor. And then phi has to be 0.85 in order to go back to a beta of 3.

These numbers are different from 0.6 to 1.17 and 1.01 that you see on the top. Likewise if we change the live load factor to 1.5 then phi needs to be reduced a little bit and that again gives me a beta of 3. Similarly if the last row is if I have to equate if the two load factors are equal gamma W and gamma Q then phi has to be brought down more to 0.65 in order to achieve the beta of 3.

So, obviously this is an undetermined problem so there are infinite possible solutions for the 3 partial factors of safety. So, obviously things are not hopeless and we are going to talk about that and we will find out how to come up with or how to reach the best possible set of these phi's and gammas for a certain application. But let us now see something interesting after understanding this problem is underdetermined.

Let us see what would happen if we now vary the load ratio W n over Q n. So, we will start with that first row 1.16 and 0.85 for the 3 partial factors which gives beta equals 3 provided W n over Q n is 0.15. And let us now make W n over Q n vary and then does the design equation need to change in order to keep the beta value unchanged at 3? The answer is yes.

But there is also a very important point to note here because once you fix the material and the uncertainties and the nominal values of the basic variables then the bias factors do not change, the COV's do not change because a material has an yield strength with the same bias and same COV, the loads whatever their absolute magnitudes are if they are of the dead load type or live load type we know what the mean is in relationship to the nominal, we know what the uncertainty is, so we know the COV.

So, those do not change, the only quantity that can change from cable to cable, from situation to situation is that W n over Q n ratio. So, what determines the goodness of a design equation is how closely it follows the target reliability with varying design situations and in this case the varying design situation is given or is expressed through that varying W n over Q n ratio. So, that is what I have circled in green in the bottom right corner.

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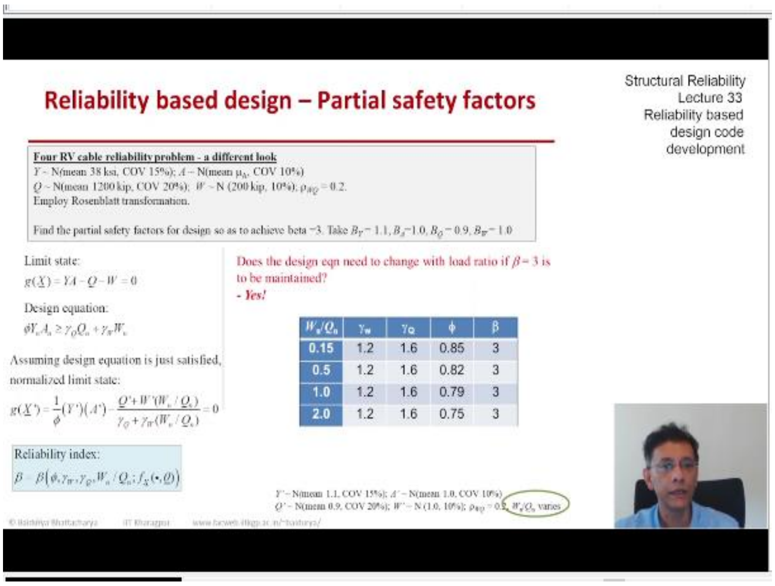

And now if we look at the results the first row is what we had in the previous slide, but then if we make W n over Q n larger at 0.5 and then further at 1 and then further at 2 we see that phi needs to change accordingly if we have to maintain the value of beta. So, again here we have kept gamma W and gamma Q fixed and now we are looking at different design situations and phi now needs to change if we have to maintain the value of beta which is 3 in this case.

Now let us flip the question. The question is that obviously we cannot keep changing the design equation; we cannot keep changing the factors. So, let us say we fix the factors and then as the design situation changes which means W n over Q n varies obviously beta will not be maintained what would happen, how badly would beta be affected would as I hinted little earlier or some sets of partial factors better than others in trying to stay close to beta as W n over Q n design situation changes. So, let us take a look at this.

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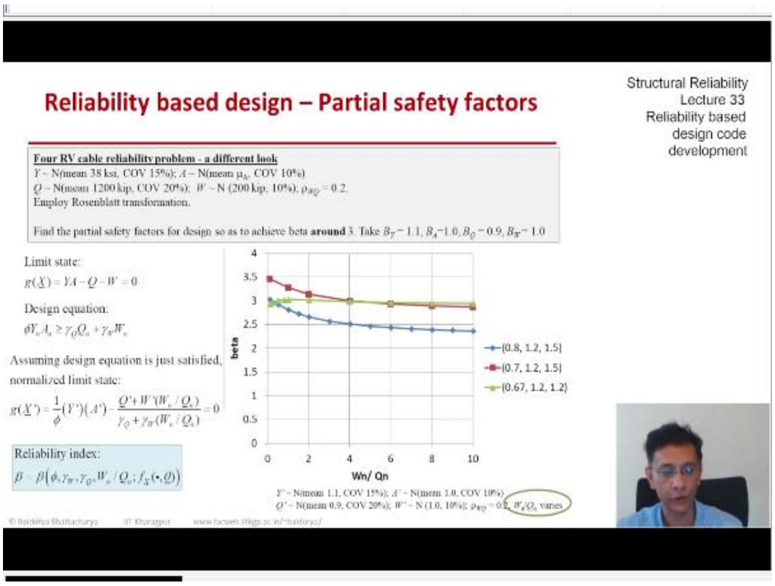

So, now as you see we do not insist that beta is exactly at 3, we are now letting the beta to be around 3. So, everything else is the same, we have the same limit state, same design equation format, we have normalized it, we understand the beta is a function of the partial safety factors, the load ratio, the load statistics and the strength statistics, but now that W n over Q n varies and that is what you see in the chart on the right.

So, we have W n over Q n on the x-axis and beta on the y-axis and we have a rather large range. So, for W n over Q n it varies from 0.1 all the way up to 10. So, 2 cycles and we can expect all design situations to be captured within this range of 0.1 to 10 for the kind of cable that we are looking at. So, now with our beta which was with our phi of 0.8 and a gamma W of 1.2 and gamma Q 1.5 earlier when it was exact when W n Q n was 0.15.

Now beta does not stay at 3 anymore for very low values of that ratio we are just at about 3 and then we fall well below 3 and we even fall below 2.5 at high values of the ratio. So, obviously this is not desirable, this for some cable for a very few number of cables I would get the target reliability but for all the rest I would be more unsafe, I would not be able to maintain that reliability.

So, this is not good, so let us try just change the factors, let us say we do not touch the load factors, let us say we just decide to touch to tweak the phi factor and so we bring it down from 0.8 to 0.7 and low and behold we actually have a much better situation under designing the low beta is not so much it kind of even for very high values of the ratio the worst that we see something like 2.8, 2.9 and for about almost half the cases for the range 0.1 to 4.

We have higher reliability than needed and maybe this would be acceptable as a design equation, but then you might start wondering on the other sets of these partial factors where I could do even better and so what we are hinting at is that this is now an optimization problem. If we can somehow cause the problem as minimizing the deviation from the desired value of 3 in this case then we actually have a well posed problem, it is no longer underdetermined.

And now we can solve as the outcome of an optimization problem the values of phi gamma W gamma Q which give the least variation, the least scattered, the least deviations from the target provided we have the tools to run such an optimization program. We are not doing that yet now, but it can be done, it is done, it should be done. Now the last example that we see in this series would be and obviously I have done many trial scenarios and with 0.67 of phi and the gammas being equal 1.2 and 1.2 which might be counterintuitive because they do not have the same level of uncertainty.

But regardless if this is acceptable then we have a designs equation which gives very, very accurate and almost unchanging value of beta. So, whatever your design situation, whatever your cable throws at you in terms of dead load and live load nominal values , your this design equation of 0.67 1.2 and 1.2 is actually going to give you a beta that is almost 3 from the lowest value of the ratio to the highest value.

So, this is quite a happy ending that we see. Now let us generalize this and see how this sort of thing is done in an actual code development or code equation development and then solve a little more realistic problem and then end this lecture.