Structural Reliability
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Lecture-234
Reliability Based Design Code Development (Part-01)

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In the two previous lectures we saw how reliability-based design codes evolved, how they are structured and in particular how partial safety factors appear in modern design equations? We worked through several examples and showed how a single factor safety or a set of partial safety factors can be derived in order to achieve a target reliability index for a given limit state.

We then said that a designer can use these design equations without bothering to know what the intended reliability is and how the factors were derived. But you might have wondered is so much mathematical formulation and reliability computation worth designing for a single cable in producing those design equations and factors of safety. We could equally have given the designer the final solution out of our reliability analysis instead of an equation.

And you might have wondered if only the design equation could work for all cables of this type. If you also remember we mentioned that these partial safety factors are intended to achieve a specified minimum reliability in design at least in an average sense. So, let us build up on these thoughts in this lecture and see if we could address that concern that a design equation should be able to work in a large number of situations for a given class of problem.

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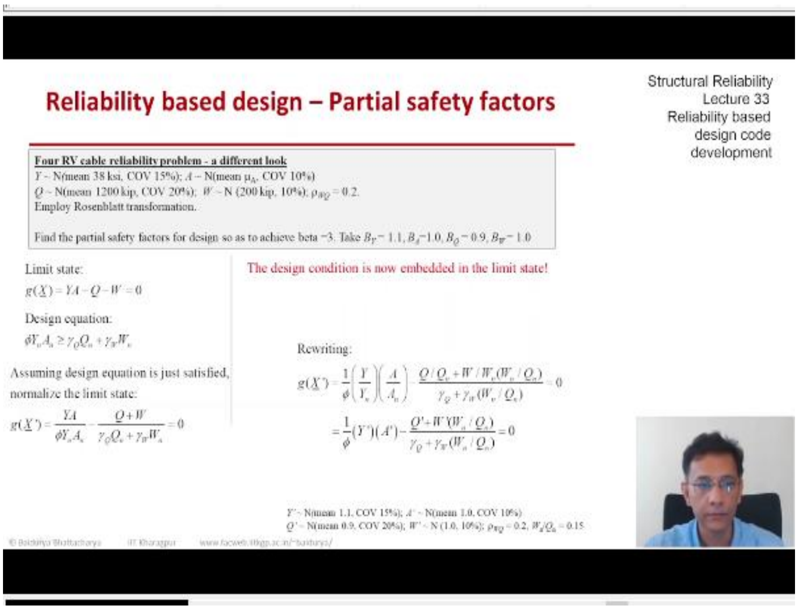

It is helpful if we start with an example and we will pick up the same example that we solved last this 4 random variable, cable reliability problem in which there were 2 loads and we went through a form exercise first order reliability method and came up with a design equation corresponding to a required beta of 3. So, if you wish you may please go back and take a look at this example but we are now going to build up on this.

So, this was our limit state, yield times area Q - W is 0. So, yield times area - live load - dead load is 0. So, that is our limit state equation and let us say this is our design equation; we have already looked at this we have presented one set of factors for this equation. So, this is known to us. So, the design equation is a factor 5 which we can call the resistance factor or material factor times the nominal yield and the nominal area and that should be at least equal to the sum of the factored dead load effect and the factored life load effect.

Obviously if non-linearities were involved we would do the factoring first and then do the analysis, but here we are just looking at the linear combination. Now here is a very important step and what we will do is assuming that the design equation is just satisfied, so phi Y n A n = gamma Q Q n + gamma W n and subscript indicating nominal values as you remember . Then we will normalize the limit state. So, because now the 2 sides are equal of the design equation.

We will just divide the limit state the capacity part with the left hand side and the demand part with the right hand side and come up with an essentially same limit state equation but normalized. So, what you see is g in terms of x prime not x where you have the same random variables as before but the additional terms Y n A n phi and Q n W n and gamma Q and gamma W.

Now this will actually prove to be quite helpful, let us first rewrite this we will club terms together, so Y over Y n; A n we put together and then the load side we just do a few more algebraic manipulations. So, we divide both the numerator and the denominator by Q n. So, that is why you see all those terms involving Q n in the denominator, so we have Q over Q n. We also have W over W n, but then that is factored by the ratio of the 2 nominal loads.

So, W n over Q n and that is actually significant and then in the denominator we have the partial load factors of gamma Q and gamma W, but gamma W is modified by the nominal load ratios. So, effectively what we have is an equation now we can simplify we have already hinted. So, we have x prime instead of x, so each member of X prime is Y prime, A prime, Q prime and W prime and they are respectively normalized by their nominal values.

So, let us first look at the statistics of these primed random variables and we see that effect of embedding the design condition in the limit state gives me some very interesting results. So, this Y prime is now a normal random variable just like Y is its COV is 15%, its mean is not 38 its mean is 1.1 which is its bias factor, likewise A prime has a mean equal to its bias factor. It is means A's bias factor, the same COV as A Q prime has a mean of 0.9 which is the bias factor of Q.

W has a bias factor 1, so W prime has a mean of 1 and the COVs are the same respective values not only that the row between W and Q is the same as that between W prime and Q prime. That we know very well and there is an additional term that the nominal load ratio W n over Q n, so that happens to be 0.15. So, we are now left with by embedding the design condition in the limit state, we are now left with limit state equation that involves random variables with normalized quantities. So, all these random variables are unitless and their mean is the bias factor which actually is going to be quite significant as we see next.

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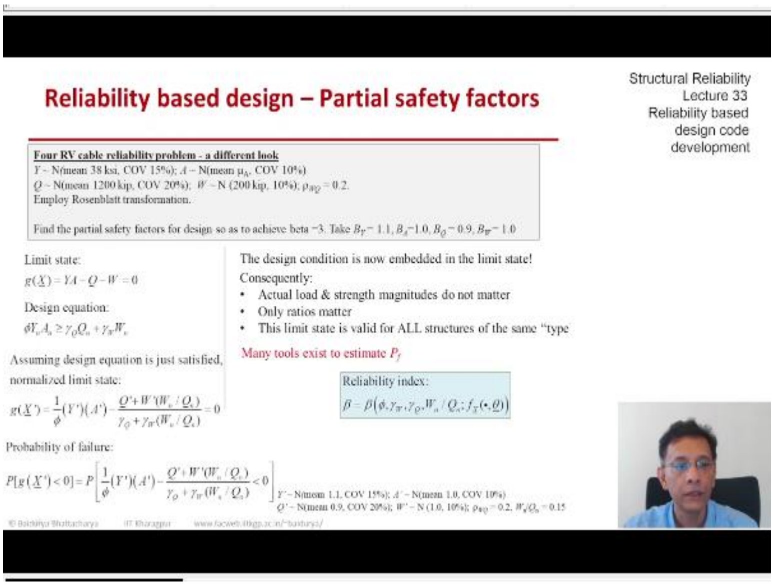

So, let us see what that would be, so on the left we have the steps that we are taking so we have the limit state as you see on your screen, the design equation, the normalized limit state in terms of the X primes and now let us go through the consequence of what we did one by one. So, because we normalized it this way and g is now in terms of X primes the actual load and strength magnitudes do not matter anymore.

What matters are the ratios, so which ratios are we talking about? The bias factors and the nominal load ratios W n over Q n. So, the 2 loads the nominal dead load and the nominal life load ratios. So, this limit state is valid for all structures, so this is what is actually significant, it is valid for all structures of the same type, type under quotations so what do we mean by that?

We mean that as I have circled in green at the bottom and yield strength which is normally distributed whose mean is 1.1 times its nominal, it does not matter what the actual mean is, it could be 38 ksi as we had before, it could be 60 ksi. As long as its bias is 1.1 and uncertainty the COV is still 15%, this limit state would work. The same goes for A prime, the same goes for both the loads. So, it does not matter what the exact live load magnitude is as long as it is a normal random variable with a bias of 0.9 and COV of 20% this limit state works.

And equally importantly it does not matter what the absolute magnitude of the loads are or the mean loads are, as long as the nominal ratio is W n over Q n is 0.15, this limit state would work. So, that is why I said that this limit state is valid for all structures of the same type. So, now if this limit state gives a certain set of factors we could start hoping that it could work for all cables or all structures which has this same type as I described.

So, to put these things together let us now explicitly talk about the probability of failure and reliability. So, the probability of failure is P of g X prime less than 0 just like any limit state and that can be given in terms of all the phi gamma, gamma Q and gamma W numbers and the statistics of the prime quantities and the nominal load ratio and so we know that there are we have used many tools to estimate this P f. We could use form, we could use Monte Carlo Simulations, we could use important sampling, we could do more if we had more tools, but we know how to find this P f?

So, if we know P f we can take the phi inverse of that and come up with the reliability index which is a function of the partial safety factors, the W n over Q n and all the statistics of the X prime. So, what you see that vector theta in the end is basically all the statistics of the X primes. So, we can now find this beta whether through form as I said or any other means and we would need to do a phi inverse if our target has been given in terms of a beta value.

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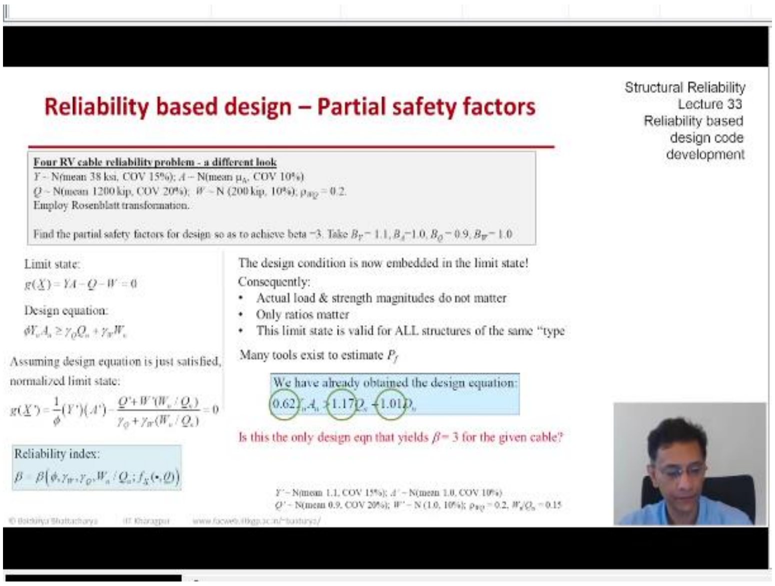

So, then the question which would come next is this reliability index that we now can find and we already have obtained the design equation for this cable where the phi factor was 0.62, the gamma factors were 1.17, 1.01 as we did in the last lecture. The question is that is this the only design equation by which we mean or those that the only 3 factors that would yield beta equals 3 for this given cable.

You might already know the answer, you might have guessed the answer, but let us try to answer this question in the next few slides and see whether an equation like this is the only one that gives me beta equals 3 or there are many other possibilities that would give me the same beta of 3, I mean so then which set are we going to choose.