

Structural Reliability
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Lecture-233
Reliability Based Partial Safety Factors (Part-03)

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Reliability based design – Partial safety factors

Structural Reliability
Lecture 32
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Four RV cable reliability problem
 $F \sim N(\text{mean } 38 \text{ ksi, COV } 15\%)$; $t \sim N(\text{mean } \mu_t, \text{COV } 10\%)$
 $Q \sim N(\text{mean } 1200 \text{ kip, COV } 20\%)$; $W \sim N(\text{mean } 200 \text{ kip, } 10\%), \rho_{QW} = 0.2$
 Find the mean cable cross-sectional area in order to achieve $\beta = 3$.
 Employ Rosenblatt transformation.

Find the partial safety factors for design. Take $B_f = 1.1, B_r = 1.0, B_Q = 0.9, B_W = 1.0$

$X_1 = F(\mu_f, k_f), X_2 = t(\mu_t, \sigma_t), X_3 = Q(\mu_Q, \sigma_Q), X_4 = W(\mu_W, \sigma_W)$
 $g(\underline{X}) = X_1 X_2 - X_3 - X_4$

min $\underline{\mu}$
such that $h(\underline{\mu}) = 0$

Solution


$\mu_t = 80.8 \text{ sq in}$

$\underline{\mu}^* = -2.38, -1.15, 1.40, 0.14$
 $\beta = 3.00$

$\phi_k = B_k(1 - \beta V_k \alpha_k)$ for k strength type variables
 $\gamma_k = B_k(1 - \beta V_k \alpha_k)$ for $m - k$ load type variables

$\alpha = 0.79, 0.38, -0.47, -0.05$

$\phi_f = 0.71$
 $\phi_t = 0.88$
 $\gamma_Q = 1.17$
 $\gamma_W = 1.01$



Let us take up a 4 random variable problem and derive the partial safety factors for the corresponding design situation. It is the same cable reliability problem that we have been looking at in various different ways throughout this course. So, here we have the yield strength, the cross-sectional area, the load Q and the load W being the 4 random variables we can think of Q as the live load and W as the dead load.

And there is a little bit of dependence between Q and W. They are all normally distributed, we need to find the mean cable cross sectional area that is the only unknown or only degree of freedom that we have and we would like to achieve a beta of 3. Corresponding to that we then would like to find the partial safety factors to be used in our design equation. The additional information that we need because we are going to use nominal quantities in the design equation we need the bias factors.

So, they have been given in the last line of the problem statement. Since there is a dependence involved and we do not want to lose that information we will employ the Rosenblatt transform. As you see unlike in the previous cases all the 4 random variables are

normally distributed and the reason we did that is because we would still like to stick with form in these PSF example and we would like to make use of the expression of the partial safety factors in terms of the bias factors, the coefficient variation, the reliability index, the sensitivity and so on.

So, the problem is formulated as before; the 4 random variables are X_1 , X_2 , X_3 and X_4 and they have been defined on your screen and then the limited equation is $X_1 + X_2 - X_3 - X_4 = 0$. And we map it to the independent standard normal space and obtain the minimum distance to the transformed limit state which is again we have done that many times so we are not going to go through those steps but we need to ensure the reliability of β of 3.

So, that gives us a condition for the unknown mean of A and that solution let me present it if you want to do it yourself please pause the video, otherwise the solution is about 81 square inches. So, a mean of that is going to give us a reliability beta of 3 given all the other statistics of the problem. So, if we now need the partial safety factors for design we would need the checking point, we would need the sensitivities and so on.

So, this is the other details of the solution, so we have the u^* values for the 4 random variables 1, 2, 3, and 4 and if you square them and sum them you get a value of 9 roughly. So, we need now to find the partial safety factors from these equations that we derived in the previous slides. So, the B values are given. The bias factors for all 4 of them we have beta which was given 3.

The coefficient of variations has also been given in the problem statement and the alphas are what we need and the alphas are defined as negative of u over beta. So, that gives us the 4 alpha values and if we now use them in defining the safety factors then we have the 4 safety factors, the 2 with the strength type quantities 0.71 and 0.88 and the 2 with the load type quantities of 1.17 and 1.01. It is all quite consistent with our expectations, the load W has lower variability, its COV is 10% compared to that of Q whose COV is 20.

So, we would expect a higher partial safety factor with Q than W and that is what we see here and yield has a COV of 15% and A has a COV of 10%. So, the yield partial safety factor is lesser than that in that for area so these all agree with our intuition and the load type partial factors are greater than 1 which is also consistent with our intuition. And as well as those

with the strength type are less than 1 which is something we are used to. So, now we are ready to provide the design equation.

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Four RV cable reliability problem
 $Y \sim N(\text{mean } 38 \text{ ksi, COV } 15\%)$; $A \sim N(\text{mean } \mu_A, \text{COV } 10\%)$
 $Q \sim N(\text{mean } 1200 \text{ kip, COV } 20\%)$; $W \sim N(200 \text{ kip, } 10\%)$; $p_{\text{target}} = 0.2$
 Find the mean cable cross-sectional area in order to achieve $\beta = 3$.
 Employ Rosenblatt transformation.

Find the partial safety factors for design. Take $B_f = 1.1, B_r = 1.0, B_Q = 0.9, B_W = 1.0$

$X_1 = Y(\mu_Y, \delta_Y), X_2 = A(\mu_A, \sigma_A), X_3 = Q(\mu_Q, \sigma_Q), X_4 = W(\mu_W, \sigma_W)$
 $g(\underline{X}) = X_1 X_2 - X_3 - X_4$

min $\mu^* g$
 such that $h(\underline{\mu}) = 0$

Solution


$\mu_A = 80.8 \text{ sq in}$

$\underline{\mu}^* = -2.38, -1.15, 1.40, 0.14$
 $\beta = 3.00$

$\phi_Y = 0.71$
$\phi_A = 0.88$
$\gamma_Q = 1.17$
$\gamma_W = 1.01$

Limit state is separable into C and D:
 $C = X_1 X_2$
 $D = X_3 + X_4$
 $C_d = 0.62 Y_n A_n$
 $D_d = 1.17 Q_n + 1.01 D_n$
 Design equation:
 $0.62 Y_n A_n > 1.17 Q_n + 1.01 D_n$

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So, let us do that with those 4 partial factors and it is good though the way the problem is defined is the limit state can be separated into capacity and demand type variables. So, let us define C as X 1, X 2 and D as X 3 + X 4. So, we have a nice C - D representation now using the definition of design C d, the design capacity it is 0.71 times 0.88 that is about 0.62 times the nominal quantity.

So, we have effectively one strength partial factor 0.62 times Y n A n the 2 nominal quantities and the design demand is the sum 1.17 Q n + 1.01 D n and if all the algebra is correct then C d and D d should be equal, the design equation is given in terms of the partial factors and the nominal quantities that the left hand side has to be at least equal to or greater than the right hand side. So, that is the design equation for this 4 variable problem and these factors are tuned to a target reliability index of 3.