Structural Reliability
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Lecture-232
Reliability Based Partial Safety Factors (Part-02)

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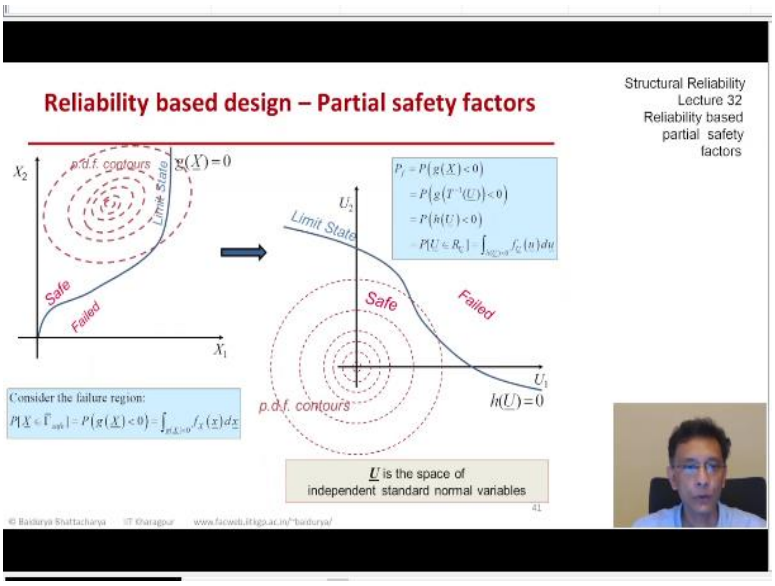

After seeing how we can develop a design equation in terms of nominal quantities and a factor of safety in the two-variable case where the factor of safety is tied to a specified target reliability provided we know the basic statistics like the mean, the coefficient of variation, the bias factors and so on. Let us now move on to the more general n dimensional case and we will continue with the form based approach the first ordered liability method based approach for a while until we abandon that for more general situations.

So, here what you see is something we have discussed a lot in the past; we have the space of basic variables X and from there we map to the space of independent standard normal variables, the limit state g X = 0, maps to the limit state h of u = 0 in the u space and the failure region likewise maps from the x space to the u space and the failure probability which is P of g x less than 0. Now because of this transformation is the probability content of some other region in the u space, there are quite a few possible maps from x to u and we have discussed those at length.

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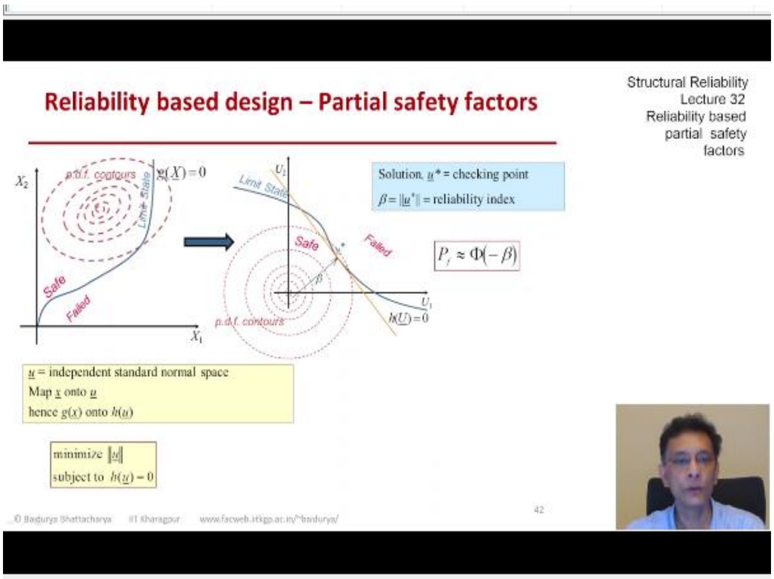

And in the end the objective is to in this u space find the minimum distance from the origin to the transformed limit state surface which is h of u = 0 and so this is the mathematical description of the problem, we minimize the distance from the origin to the line or the surface h = 0 and the solution is U star and that minimum distance is beta the reliability index.

The resultant failure probability is phi of minus beta, where phi is the normal distribution function and that approximation is caused first by the linearization at u star, so that orange line at u star that you see is the first order approximation and another potential source of approximation is the nature of the map from x to u. So, if we lose some information there then the estimated probability of failure would be an approximation.

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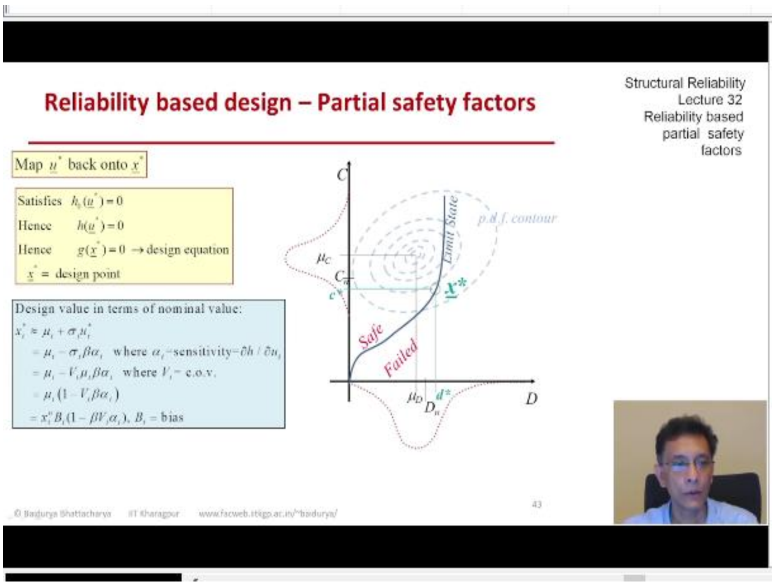

However, the important point to note here is that if we can map from x to u through a one-to-one map, we can always go back from u to x and in particular then we can map this u star this minimum distance point back onto x space and call it x star. So, u star satisfies h 0 = 0 hence it also satisfies h = 0 and hence it also satisfies g of x star = 0 and we will call this the design equation. In the diagram on the right you see the x star on the limit state, line in the x space and we actually looked at this at length when we discussed that two variable problem involving C and D.

And there we looked at the mean, the nominal and the factored quantities and so on. So, this is the same representation in the Cartesian space, but coming back to this design point and g of x star = 0 it can be quite helpful and let us see how. First we can express the design values, the design point values x star in terms of the nominal values of these random variables, again the nominal values are the more commonly known handbook values, characteristic values.

So, it might be more practical to recast the design equation in terms of the nominal quantities and that Is what we are going to do. So, the first step is to recognize that the x star is approximately a linear function of the u star and this is quite appropriate if we had used second moment transformation the (()) (06:04) transformation even if we did not this would still be an approximation worth considering.

So, if x star is mu + sigma u for each i then we can express each u star in terms of the reliability index beta and the sensitivity alpha which is partial h, partial you, so it gives an estimate of whether h goes towards failure or away from failure with a unit increment in that particular random variable. Now so we use the fact that u star is minus beta times alpha for each i.

Now then we can do some more algebra we can bring in the coefficient of variation V i instead of sigma and then we can take the mean common in that expression and that takes as mu times 1 - V beta alpha for each i and then we can bring in the bias factor, bias being mean over nominal. So, the mean is bias times nominal and finally we have an expression of x star in terms of its nominal quantity times a factor which includes the bias, the reliability index, the uncertainty V and the sensitivity alpha.

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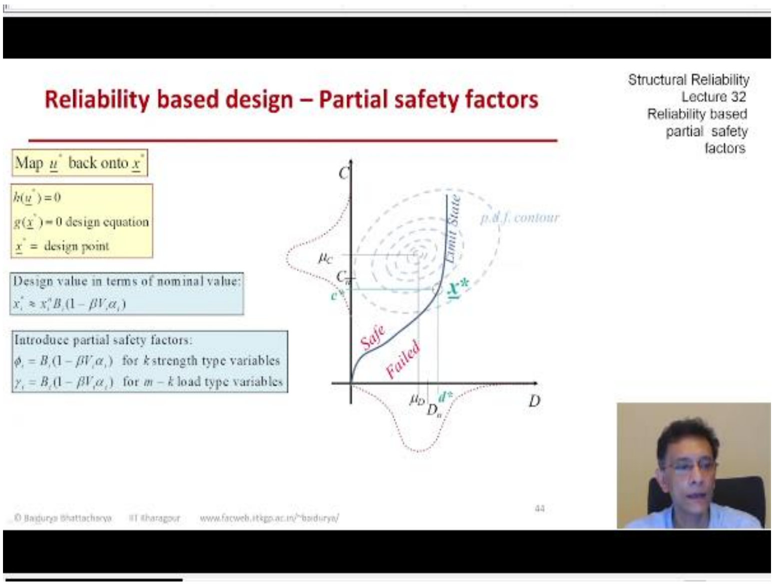

So, with these things under our belt we can now introduce the partial safety factor. So, we call that multiplier B times 1 - beta V alpha for each of these i's phi for the strength type variables and gamma for the load type variables. This is not exactly the same way that these are named for example in different codes like when we described EN 1990 the structural euro codes it was gamma. All through but here it is better to give different names to strength type and load types. So, that is why we have brought in phi and gamma for the 2 different types of partial safety factors. But in principle they are the same.

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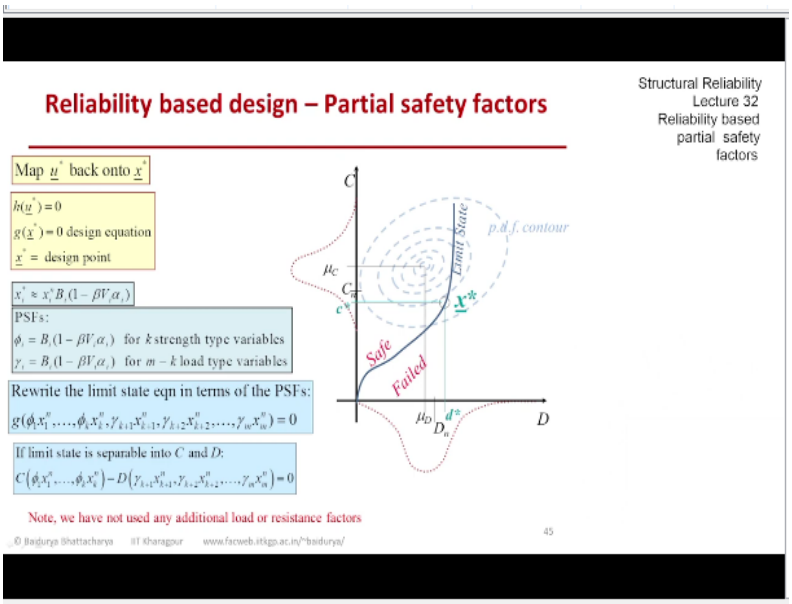

Now we can then with these partial safety factors rewrite the limited equation that g of x star = 0 at the design point in terms of these partial safety factors and the nominal values. And this actually takes us closer and closer to a design equation. So, even if we stopped here we would have an equation involving the partial safety factors, the nominal values and if the designer wanted to use this equation to find out some unknown section, some unknown geometry or any other unknown quantity this equation would actually show the way.

Now if in addition to this if the limit state is separable into C and D type variables if we can have a bunch of x's defining explicitly C. So, all the strength type variables those k that we mentioned can give C the capacity and all the other m – k the load type variables can give the demand or the load then we have a more traditional looking design equation C - D = 0. And there all the phi's and all the gammas and all the nominal quantities are listed. The point to note here is that we have not used any additional load resistance factors.

For example we do not have a different phi multiplying C which is what we have seen in the (()) (10:53) type examples. We could have that sort of formulation it would just look different but we could use that as well as a design equation. But we will come to those finer points and differences later. Another important point is if this design equation is just satisfied then the reliability achieved would be the beta that we have already used in defining the phi’s and the gammas.

Now whether that beta is acceptable or not is obviously a different question. So, if we want to achieve a certain beta like we have done in the previous examples then we might need to change tweak the phi and gamma values. And that is actually at the crux of partial safety factor based design and derivation of these partial safety factors. Let us take up a 4 variable example and see how we can see these points from that example.