Structural Reliability

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Lecture-231
Reliability Based Partial Safety Factors (Part-01)

In this lecture we are going to discuss partial safety factors used in reliability based design of structures. In the previous lecture we saw how reliability based design codes evolved? How they are structured? And in particular how partial safety factors are used to modify the nominal or characteristic values of loads in a given load combination after which the structure is analyzed and the design load effects and different components are obtained.

The design load effect is compared with the design resistance, the design resistance is the nominal resistance multiplied by a resistance factor. The nominal resistance for the failure mode in question is computed with the help of characteristic material properties which in turn have been modified by material partial factors. In LRFD load and resistance factor design, the material partial factors are not separately specified.

Now all these factors are designed to achieve a specified minimum reliability in design. At least as we will see later in an average sense. So, in the previous lecture we also worked through an example and showed how a factor of safety can be derived in order to achieve an explicitly stated target reliability index. We will build up on that thought in this lecture and subsequently we will also prove the idea that I just eluded to partial safety factors to achieve a target reliability in design in an average sense. So, let us set the stage.

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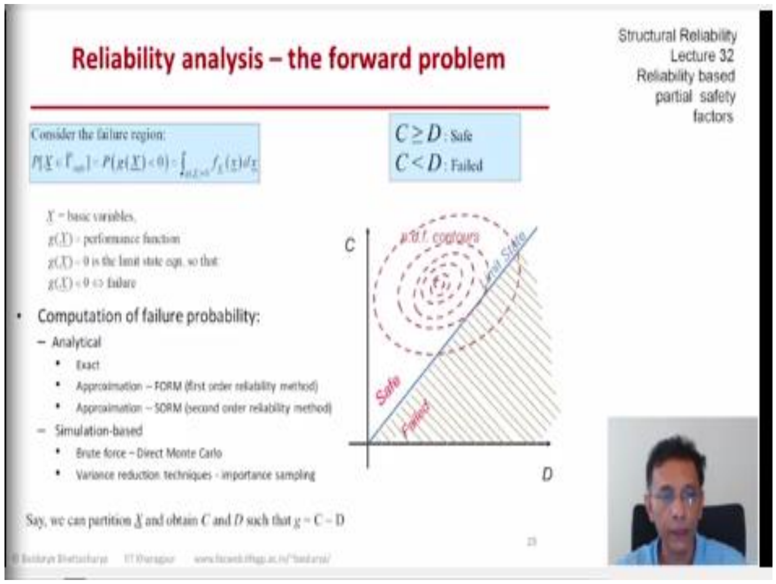

We have basic variables X and we have the limit state g X = 0 the limit state equation, so that g X negative implies failure. And we have spent a good amount of time discussing how to compute the failure probability using analytical methods, approximate methods FORM, SORM, Monte Carlo based methods and so on. So, let us say to make things simple, we can partition X the basic variables and obtain 2 quantities 2 random variables C and D such that g can be expressed as C - D.

So, it is the most simple linear combination of 2 random variables, that you see on the right part of your screen C = D is the limit state and C less than D is the failure region and C greater than equal to D is the safe region. So, that is what has been marked on the figure. Now let us say we continue with the idea of FORM first order liability method.

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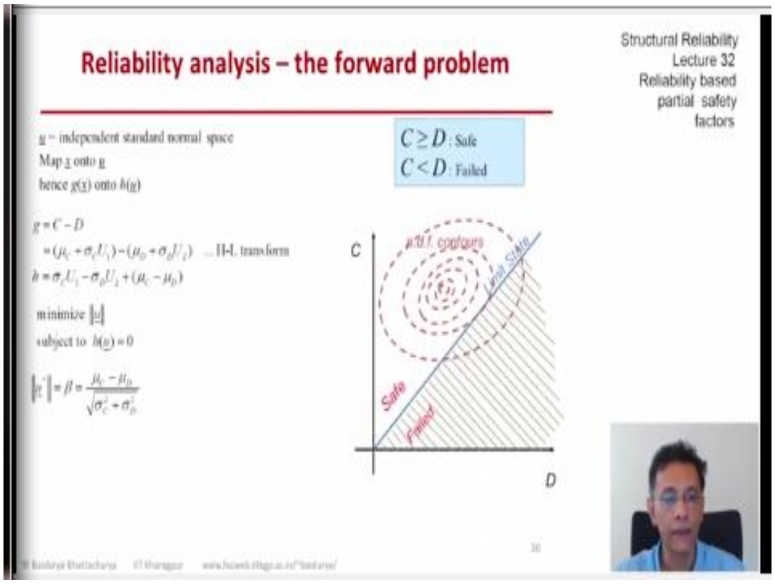

And we have the independent standard normal space u, so we map the basic variables onto this space of u. And hence the limit state function g onto h, so the new limit state is g = 0 becomes h = 0. And let us say we employ the simple second moment transformation or the Hassle-free for transformation, so g = C - D becomes a linear function in h. So, C is mu of C + sigma C times U 1, U 1 being the standard normal random variable and D is mu D + sigma D times U 2.

So, that gives me h the limit state function in the standard normal space as a linear combination of U 1 and U 2 as I just mentioned. So, it is quite trivial to find the minimum distance to this straight line from the origin we have done it many times earlier in this course. So, the answer as we know is u is mu C - mu D the difference of the means divided by the overall standard deviation the square root of c of sigma C squared + sigma D squared. This minimum distance as we know very well is also known as the reliability index beta.

So, now since we have this very simple representation of C and D and presumably they are described in the same units. So, we instead of having a Cartesian plane of C and D we can put them side by side on the same axis and let us do that.

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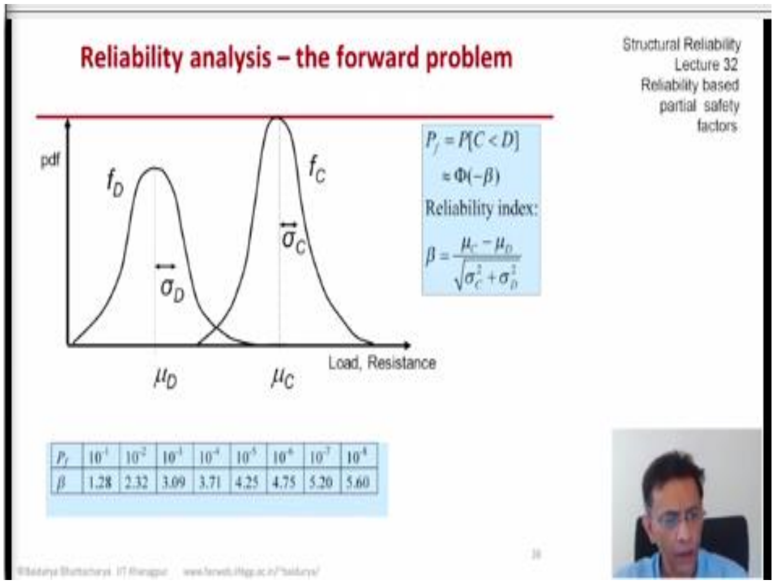

So, this would be comparing C and D side by side and the failure probability is P C less than D and we see the 2 density functions of C and D side by side. D is obviously to the left of C because we want the failure probability to be low enough. What I have marked here on the 2 density functions is the mean of D and the mean of C respectively and the standard deviation of D and the standard deviation of C, just to show the relative scatter the spread in the 2 distributions.

And I have also shown on the top right corner the reliability index which we already derived in the previous slide as the ratio of the difference of the mean and the composite standard deviation. Now on the lower part of your screen you see the relationship between failure probability and beta and for components and systems in our structural engineering. We typically see beta values between say 1.5 and 4, 4 and a half.

So, that is the numerical range in which we find most of our components and systems to **to** fall. Now this is so far the forward problem in reliability. So, we have been given a structure and if we know it is strength, if we know the load, if we know all the distributional properties we can find the reliability of the structure. So, that is the forward problem and it is a relatively easier problem. The inverse problem is when things become more interesting.

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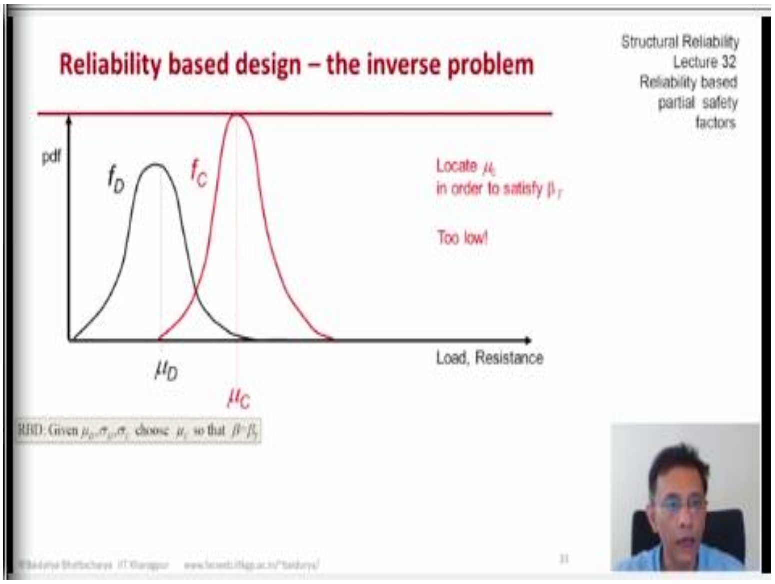

And this is the statement of the inverse problem. We have been given a load, so I have not changed the distribution of the load; it is unchanged from the previous slide. But now the distribution of C is missing, because that is what we need to find out. And so the reliability based design inverse problem is given mu D, sigma D and sigma C, choose the unknown mean of C. So, that beta equals beta target.

So, in effect what we are saying is that given the load including it is mean and all uncertainties there is nothing we can do about it. We also acknowledge the fact that C has uncertainty, the capacity we cannot eliminate that, so we have to live with that. What we can do is we can locate the distribution of C. So, in effect we can go for a stronger or weaker or just strong enough capacity, so that the reliability index is just satisfied.

We do not want to spend too much money and over design and obviously we do not want to under design. So, this is the problem statement. So, let us say that we choose some mu of C and we find as you see on your screen that is where the density of C appears in respect to the density of D. And clearly they are too close to each other, so we can do the computation or we can just do it by inspection. And say that it this is too low, this is not going to work and this is basically an identity design process that is commonly done.

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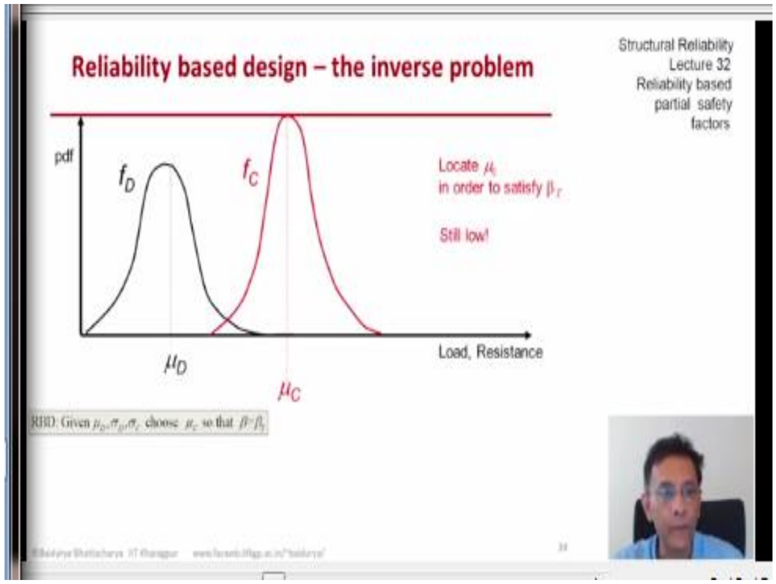

So, maybe we will move a little bit to the right and again do the computations and we will find out if things are better but it is still low.

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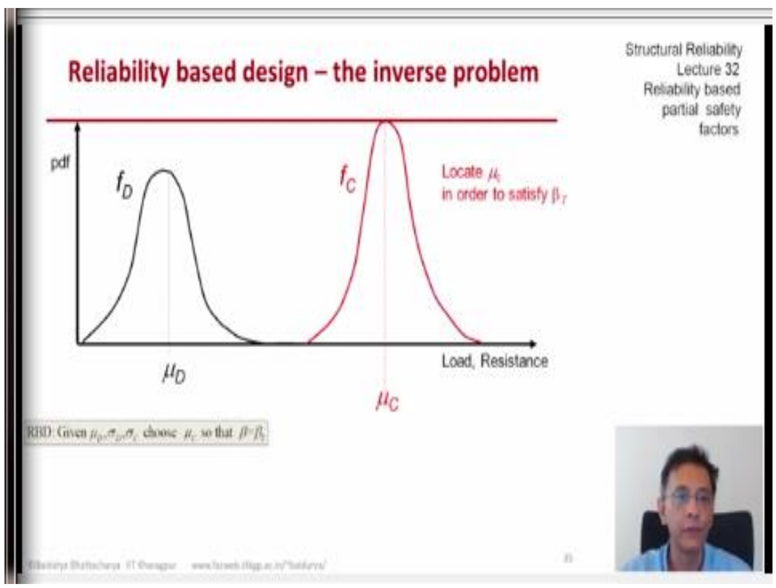

So, we will move further to the right, we will find that in this situation maybe we have over designed, so this is too high. If we go for this solution then we will be unnecessarily wasting resources. So, we can do this but keep doing this until we will converge.

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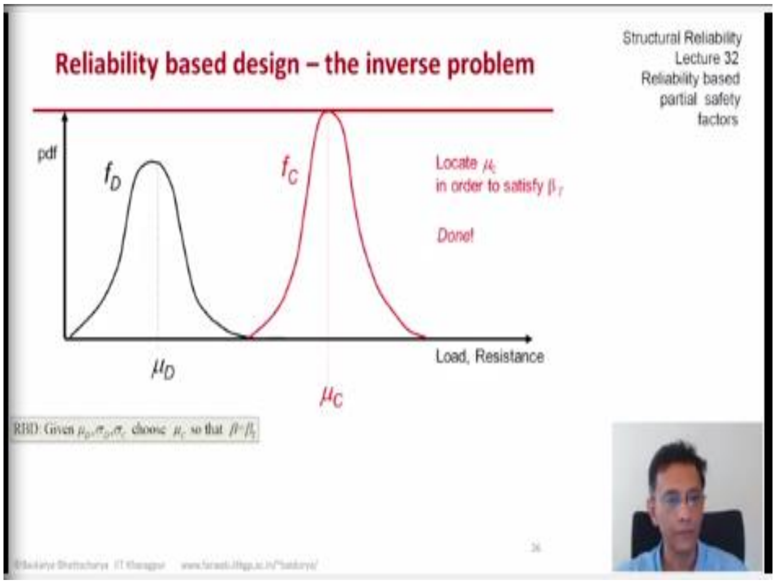

And let us say we converge in this situation, so this is just right this is done. So, once this is done we know the solution, it is the mean of C is the mean of D + quantity that depends on the target reliability index and the total variance. So, it is mu D + kappa times beta t where kappa square is sigma D square + sigma C square. So, this would be the end of it, we have solved the inverse problem but because we want this to be used in the design office. Later on we want to present this solution in more familiar way.

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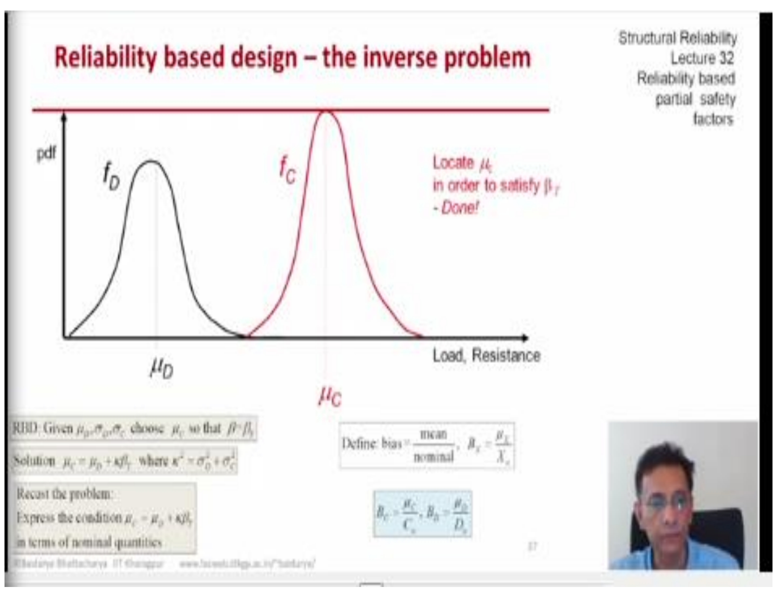

So, let us say now we have been asked to give the same answer but we need to express the condition that mu C is mu D something in terms of the nominal quantities of C and D. So, we have mentioned this term nominal quantities quite a few times in this lecture and in the past one. So, let us define them, so what we have is let us say we have the ratio mean over nominal.

Depending on whether we are talking about a strength type quantity or a load type quantity the nominal or characteristic quantity typically is less than the mean value for strength type quantities and greater than the mean value for load type quantities. So, that ratio is expressed as the bias in that nominal value, so the bias is the mean over nominal. And we often use the letter B for this quantity bias.

So, just like earlier we defined the coefficient of variation sigma over mean. So, now we have mean over nominal which is the bias or bias factor. In many applications we define the characteristic value of a strength type random variable as a very low percentile in the distribution, for example the 5th percentile value. So, if it is a normal distribution the 5th percentile corresponds to 1.645 standard deviations below the mean.

So, that let us define this bias in terms of the mean and the COV of the distribution. Anyway, so for this particular purpose since we need to bring in the nominal quantities and the means have been used in the derivation, we need the 2 bias factors. So, B of C and B of D which we are going to use in the next few slides.

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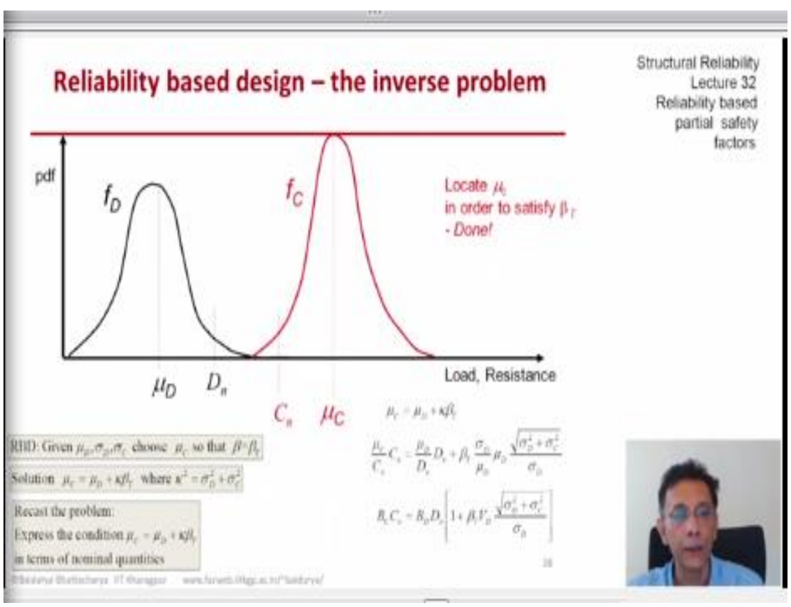

So, let us try to move towards the desired solution. So, mu C is mu D + kappa times beta target and now we just keep do some simple algebra. So, we define the mean of C in terms of the bias of C and the nominal value of C, likewise we do the same for D. And in terms of kappa what we do is that we bring in mu D and sigma D both in the denominator and numerator but with a certain purpose in mind which will be clear in the next equation.

When we bring in the D n and C n values, so you see in the graph we have D n which is the nominal value of the demand, the load. So, that is to the right of mu D and you see C n the nominal value of the capacity which is the left of mu C. And now bringing in the bias factors we have the bias of C times the nominal of C n is equal to the bias of D times the nominal of D times a factor which depends on the target reliability.

The COV; the coefficient of variation sigma over mean for D and a quantity which is basically the ratio of the composite standard deviation and the standard deviation of D. We have actually come across this term earlier when we described FORM.

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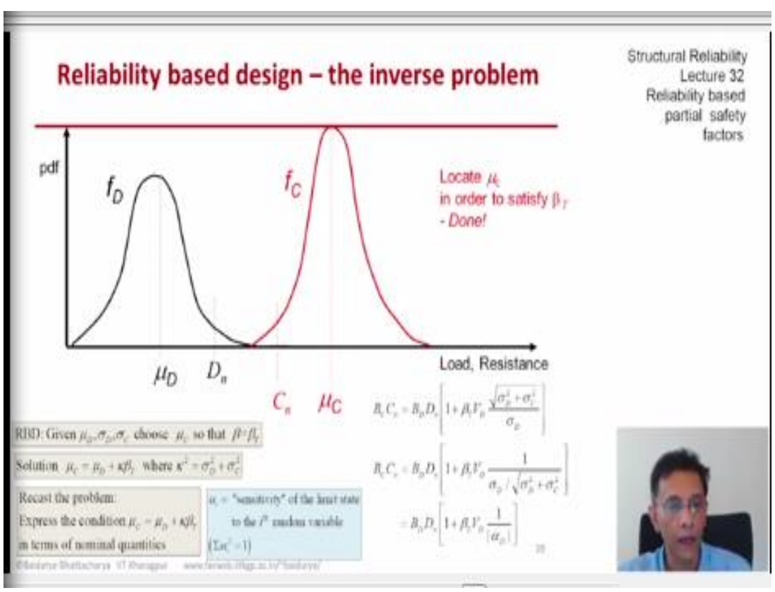

This ratio of the sigma D the standard deviation in the load and the composite standard deviation is the sensitivity factor. So, what we do is we bring in the alpha D the sensitivity factor of the random variable D. So, that is basically the partial derivative of g, the limit state in U space with respect to the ith variable. So, alpha i is del h del u i, so it is a sensitivity of the limit state to the ith random variable, so that the alpha squares sum to 1. So, now we are left with an equation involving the nominal C, the nominal D and certain factors like the bias, the target reliability, the coefficient of variation and the sensitivity. So, let us put all of this together.

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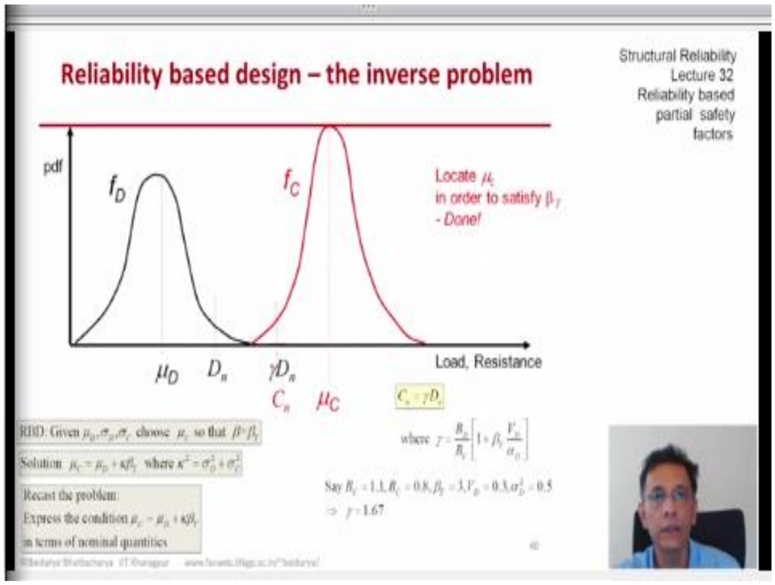

And we realize that if we just rename those quantities and put all of them into a single factor gamma then we actually have arrived at a relationship C n = gamma times D n where gamma is an expression involving the ratio of the bias factors and a product of a term involving the target reliability, the uncertainty in D and the sensitivity of t. Now let us put typical values just to see how much sense this would make.

So, in the graph I just introduced as you see if you look at the graph I have mu D, I have D n to the right of it, I have mu c far to the right of mu of D C n is a conservative quantity, so it is an underestimate, so it is to the left of mu C. And then because of this relationship that I just derived I have gamma D n exactly equal to C n. So, that is 1 single relationship between the 2 quantities D and C which if satisfied would guarantee the target reliability of beta t.

And now putting the numbers let us say the bias in C is about 1.1, so the mean over nominal is 1.1 the bias in D is say 0.8 there is a mistake there, it is B of D 0.8. The target reliability is 3 which we have commonly been taking, the uncertainty in D is 30 let us say the 2 variances of C and D they are roughly of the same value. So, sigma square D would be about half. So, all of that if we just put in the numerical values we get a gamma of 1.67.

So, again we come to this quantity 1.67 as another coincidence, we have seen this number before. But this would be a way in which we can derive a design equation where we use the nominal values the well known nominal values that we are very familiar with for the quantities D and C. And a factor that pulls in everything together and is designed to ensure a intended target reliability of beta target which was 3 in this case.

So, now next we are going to see how this can be generalized when there are more than 2 random variables involved. So, we are going to have just not 1 safety factor like we have been doing but more than 1, so 1 safety factor with each random variable that we will see and with the idea that a pre specified target reliability will be satisfied.