

**Structural Reliability**  
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**Lecture-227**  
**Reliability Based Design Codes (Part-03)**

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### Recap: Capacity Demand example with FORM

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**Example B1: two RV cable reliability problem**

Yield strength,  $Y \sim N$  (mean 38 ksi, COV 15%) Axial load,  $Q \sim N$  (mean 100kip, COV 10%)  $Y$  and  $Q$  are independent.

Find the cross sectional area of the cable if the target reliability index is 3.

Choose  $M = g(Y, Q) = g(\underline{X}) = aY - Q$   
 $X_1 = Y, X_2 = Q$

Choose  $T: u = (x_1 - \beta) / \sigma$

Then  $h(u) = a\sigma_1 u_1 - \sigma_2 u_2 + a\mu_1 - \mu_2$

Minimum distance from origin:  

$$\beta = \frac{a\mu_1 - \mu_2}{\sqrt{a^2\sigma_1^2 + \sigma_2^2}}$$

Required:  $\beta = 3$

Solving,  $a = 51 \text{sqin}$

Minimum distance point:  

$$u_1 = \frac{a\sigma_1(a\mu_1 - \beta\sigma_2)}{a^2\sigma_1^2 + \sigma_2^2} = -2.8$$

$$u_2 = \frac{-\sigma_2(a\mu_1 - \beta\sigma_2)}{a^2\sigma_1^2 + \sigma_2^2} = +1.0$$


Design point:  

$$Y^* = x_1^* = \mu_1 + \sigma_1 u_1 = 22 \text{ ksi}$$

$$Q^* = x_2^* = \mu_2 + \sigma_2 u_2 = 1100 \text{ kip}$$

Prescribing the design equation as  $aY^* \pm Q^*$   
 to designer would ensure  $a = 50 \text{ sqin}$  and  $\beta = 3.0$  without the need for first principles based reliability analysis!

Finally,  $Y^*$  and  $Q^*$  can instead be presented to designer in terms of factored characteristic values



We go back now to a problem that we solved earlier in this course in the context of discussing FORM first orders reliability method. And see how that simple reliability analysis problem can be recast as a reliability based design equation problem. So, that we can satisfy a certain target reliability. So, this was the problem, we had 2 random variables Y and Q both were normally distributed and they were independent.

So, the task was to find the cross section area, so that a target reliability of 3 could be satisfied. So, this is how we set the problem of we chose the obvious form of the limit state equation  $A Y - Q = 0$  and did a simple hazard for Linde transformation from the basic variable space into the independent standard normal space  $x$  to  $u$ . And we came up with a linear limited equation in  $u$  space from which simple geometry gave us beta as a function of A.

And then we solved that quadratic equation and obtained the value of A as 50 square inch. And then we ended that example actually with the hint that we could take this development forward

and present a design equation. So, let us see what we suggested there? We got the design point back from the u space into the x space, so that was 22 ksi and 1100 kip respectively. And then perhaps we could give a design equation a  $Y^*$  star greater than or equal to  $Q^*$  star to the designer.

And then the designer would take it forward and do the fabrication and installation as necessary. So, in fact we also suggested that instead of these arbitrary design point values we could give this equation in terms of more well known characteristic values or nominal values. So, let us now see how to take that to conclusion?

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### Recap: Capacity Demand example with FORM

**Example B1: two RV cable reliability problem**  
Yield strength,  $Y \sim N$  (mean 36ksi, COV 13%). Axial load,  $Q \sim N$  (mean 1080kip, COV 10%).  $Y$  and  $Q$  are independent.  
Find the cross sectional area of the cable if the target reliability index is 3.

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Minimum distance point:  
 $\beta_1 = -\frac{\mu Y_1 - \mu Y_1 - \beta_1 \sigma_1}{\sigma_1} = -2.8$   
 $\beta_2 = -\frac{\mu Q_2 - \mu Q_2 - \beta_2 \sigma_2}{\sigma_2} = 1.0$

Design point:  
 $Y^* = \mu_Y + \sigma_Y \beta_1 = 22 \text{ ksi}$   
 $Q^* = \mu_Q + \sigma_Q \beta_2 = 1100 \text{ kip}$

Typically, a "nominal"/"characteristic"/"handbook" value is a conservative representation:  
 - an under-estimate of "strength"  
 - an over-estimate of "load"

Let  $Y_n = 36 \text{ ksi}$ ,  $Q_n = 1080 \text{ kip}$

Then,  $\alpha \geq \frac{Q^*}{Y^*} = \frac{(Q_n / Y_n) Q_n}{(Y_n / Y_n) Y_n}$

Putting the numerical values,  
 $\alpha \geq \frac{(1100 / 1080) Q_n}{(22 / 36) Y_n} = 1.67 \frac{Q_n}{Y_n}$

Design eqn:  $\alpha \geq (FS) \frac{Q_n}{Y_n}$ ,  $FS = 1.67$

So, let us put all the things we already did on the left and on the right, we are going to present the solution. So, just to recall typically when we talk about a nominal value, a characteristic value, a handbook value we mean it is a conservative representation of a random quantity. And if it is a strength type quantity the nominal value is typically an underestimation of that.

And if it is a load type quantity it is typically an overestimation of that. And we have done this sort of thing for concrete strength, for steel strength and so on. So, that something we are already familiar with. So, for this example without going into the statistics and what percentile of the distribution should be the characteristic value? Let us just say arbitrarily that  $Y_n$  is 36 ksi and  $Q_n$  the nominal load is 1080 kip.

So, the nominal load is a little higher than the mean load and the nominal yield strength is a little lower than it is mean. With those then we can just recast the design equation or the requirement that  $a$  has to be at least equal to  $Q^*$  over  $Y^*$  in terms of the nominal quantities and you see what we have done. On the screen we have the ratios  $Q^*$  over  $Q_n$  and  $Y^*$  over  $Y_n$  in the numerator and denominator.

And put in the numerical values we see that in this particular case  $a$  has to be at least equal to  $1.67 Q_n$  over  $Y_n$ . So, presumably  $Q_n$  and  $Y_n$  are well known quantities and the designer could just pick them up from some design handbook. And multiply that ratio with  $1.67$  and come up with a cross sectional area. So, a design equation like this that area greater than equal to factor of safety times  $Q_n$  over  $Y_n$ , where the factor of safety happens to be  $1.67$ .

And that in turn depends on what the implied reliability is would be served to the designer. And the designer without the need to know how all these, how the factor of safety came? What the implied reliability is? What the roles of the distributions were? How much uncertainty there was without the need to know all of that the designer could go ahead and do the construction.

And thereby we would ensure that the target reliability which is  $\beta = 3$  in this case was automatically satisfied. Now this idea of factor of safety of course has been around since the earliest days of formal design. But there is a significant difference from what we just did and what the traditional interpretation or utility or factor of safety has been. and I will come to that in a minute.

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## Recap: Managing uncertainties

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In 1849 the Royal Commission appointed to investigate the use of iron in railway bridges asked the prominent engineers of the time: "What multiple of the greatest load do you consider the breaking weight of the girder ought to be?" The answers ... ranged from 3 to 7. And when asked, "With what multiple of the greatest load do you prove a girder?" the panel responded with factors ranging from 1 to 3. The commission concluded that an appropriate factor of safety for railway bridges would be 6.

From *The Engineer in Charge*, by Henry Petrosky, Vintage Books, 1983.


Factor of safety : For mild steel in US:

Year	Yield strength (MPa)	Factor of safety	Allowable stress (MPa)
1890	197	2	97
1918	190	1.72	110
1923	228	1.83	124
1936	228	1.65	136
1963	250	1.67	152

Taken from "Design codes" by T. Galambos in Engineering Safety, Ed. Stockley, 1992

**Why factor of safety:**

- Uncertainty in loads
- Uncertainty in strengths
- Uncertainty in structural/ mechanical model/ behaviour



We have looked at these examples very early on in this course. This one from design of railway bridges in the mid 19th century Britain are this set of examples of mild steel designed in the US. You see over about 75 years the factor of safety fluctuated through 2 world wars and kind of stabilized at 1.67, incidentally it is the same factor of safety we derived for the cable example but obviously that is coincidence.

We also know that the factor of safety was used by designers and manufacturers, the owners and users to take care of in an aggregate sense, all those that were beyond their knowledge or beyond their control. But the difference is that until relatively recently the factor of safety was not tied to an explicit measure of safety. So, it would not be possible to look at any of these examples that you see on the screen now.

And answer the question that what sort of precise level of safety did these factor safety produce? The idea of tying factor of safety are more correctly in modern times factors of safety in plural with an explicit level of reliability in probabilistic terms is rather reset.. There is also another aspect here which is the factor of safety was typically applied in the context of stresses that were produced as part of normal operations, everyday working loads.

And then which were compared with the reduced strength of the structure. The hope was that the structure would be ok, under extreme loads where the structure would be asked to perform at its

limit. But the ability to analyze the structure under those extreme situations was typically not there. That started to change only in the last decades of the 20th century. We also see the beginning of the attempt to specify high values of extreme loads.

Say starting in the 1930s for example ASC suggestions to use 50 year and 100 year wind speeds in design. So, while this sort of trial and error evolution of the factor of safety was going on and an attempt to use data, to use statistical data in finding or in estimating extreme loads was going on. We see the beginning of a new thinking in structural safety and structural design taking place.

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**A little history**

- AM Freudenthal: Safety of Structures (1947), Safety and the Probability of Structural Failure (1956)

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