

Structural Reliability
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Lecture –224
Capacity Demand Systems Reliability (Part 15)

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Structural systems reliability

Cut set based system reliability formulation

- More general systems than pure series or parallel
- n (binary) component system: 2^n distinct system states are possible
- Only a few of these states cause failure -- a smaller subset is probabilistically dominant
- Challenge: identify the (dominant) minimal cut sets


$$P_{f,sys} = P \left[\bigcup_{i=1}^{n_c} C_i \right] = P \left[\bigcup_{i=1}^{n_c} \left(\bigcap_{j=1}^{n_i} g_{ij} \leq 0 \right) \right]$$

- g_{ij} is the j th limit state in cut set i
- Missing a minimal cut set under-estimates P_f

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When solving series systems, we showed how FORM can be used to compute the series system failure probability. We can do something similar for a parallel system and the general expression for a purely parallel system is what you see on the screen but one has to be a little careful. Because as we have seen if there are sequence effects, if we need to consider the limit states after there has been partial collapse of the structure.

Then all these g 's, all the individual limit states they need to be properly defined, but as long as that can be done we have the answer in terms of the multi dimensional normal CDF. If we have a minimal cut sets so it is not a purely parallel system, but there are number of cut sets, and so the system failure probability is the union probability and so then each cut set is composed of its own members, each minimal cut set, is composed of its own members.

And cut set i has, and i such members and g_{ij} is such a limited state in the cut set i . Again, we

need to make sure that any sequence affects any partial failure if that is how we can analyze the structure that so we need to analyze the structure is taken to account when defining the respective g's. Now this also gives us an idea that are there other approximate ways in defining or in estimating the union probability like what you see here p of union C i, i going from 1 to nc.

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$$P[F_{sys}] = P\left[\bigcup_{i=1}^m F_i\right]$$

- First order

$$\max_{i \in \{1, \dots, m\}} P[F_i] \leq P[F] \leq 1 - \prod_{i=1}^m (1 - P[F_i])$$
- Second order

$$\text{Lower bound: } P[F_i] + \sum_{j=2}^m \max\left\{P[F_j] - \sum_{i=1}^{j-1} P[F_i F_j], 0\right\} \leq P[F]$$

$$\text{Upper bound: } P[F] \leq \sum_{i=1}^m P[F_i] - \sum_{j=2}^m \max\{P[F_i F_j]\}$$



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So there are some very well known famous bounds that are put on these union probabilities. There are the first order bounds that you see on the screen and there are second order bounds the second order lower bound and the second order upper bound. Basically, uh, the order implies what sort of joint event we have information of probability for. So, if we have just marginal probabilities so that gives us the first order bound.

If we have pounds of F i F j f being the failure in the cut set, i etcetera, F j failure in the cut set j. So, if we have those type of second order joint probe information, then we can improve the bounds compared to the first order bounds and get an estimate sometimes these bounds can be quite good sometimes they are too wide to be useful. If you have third other probability information third out of joint probability then we can have third of the bounds and so on and so forth. And these can be referred to in the in the literature.

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Further reading:

Application of structural systems reliability theory by P Thoft-Christensen and Y Murotsu, Springer.

New methods in systems reliability by A Karamchandani, Report RMS-7, Stanford University.

System reliability and structural design by TV Galambos, in Structural Safety vol 7, 1990.

Structural reliability analysis and prediction by RE Melchers and AT Beck, Wiley.



So, to summarize we looked at structural system reliability in this last part of part C of our course we are going to next start with part D, the last part for further reading I would recommend some of these excellent books there is the book by Thoft-Christensen and Morotsu that is, ashish, Karamchandhani is PhD thesis at Stanford. The book by professor Galambos and the text book that you already have mentioned in this course the one by Melchers and Beck.