Structural Reliability
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Lecture –223
Capacity Demand Systems Reliability (Part 14)

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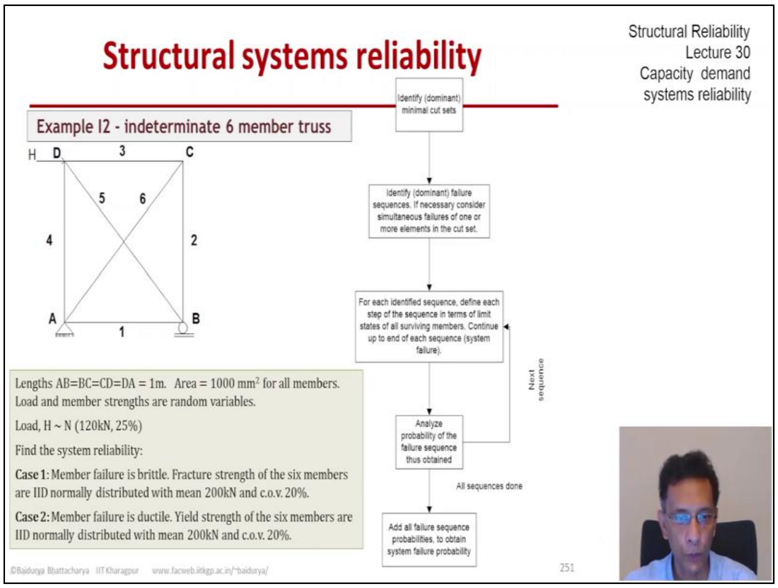

After the two unit parallel configuration let us look at a slightly more complicated structural system one degree statically indeterminate truss structure as you see on the screen. So, this system has six members and as we know very well given the sort of boundary conditions it has this is one degree statically indeterminate. So, and we can verify this that if two elements fail then this system becomes unstable.

So, and we have looked at this problem earlier in a different context where we looked at cut sets. So, this system has 15 different cut sets each of which has two members it has 15 different minimal cut sets I mean and each of those minimum minimal cut sets has two members now let's define the problem in the context of random variables and material behaviour. So, we have one load h and there are six members each of them a strength which is a random variable and we are looking at two extreme cases one is that the member failure is brittle.

So, each member has a brittle behaviour and the other one is each member is perfectly ductile and in all cases we are going to take the six random variables of strength and one for the load all seven of them are mutually independent. Now in a situation such as this we would need to identify the minimal cut sets at least the probabilistically dominant ones and then find out if we need to consider any sequence effect or not that depends on our understanding of the mechanics of the problem and how the intact structure finally goes to global failure.

And then we need to find the probability of each such cut set or sequence within the cut set and then obtain the union failure probability for this simple structure we have been able to identify all the 15 minimal cut sets but it does not have to be. So, for a larger more complicated structures but let us continue with this and let us look at case one first where the members are all brittle.

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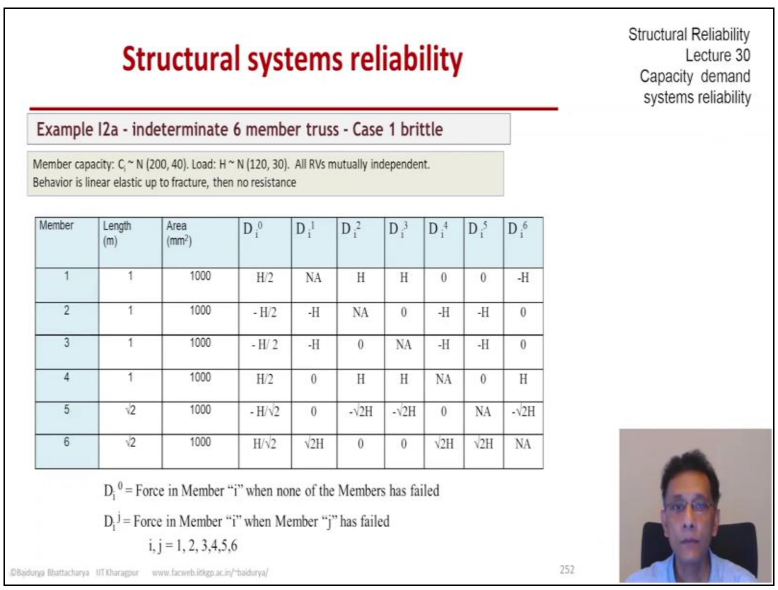

Member	Length (m)	Area (mm²)	D_i^0	D_i^1	D_i^2	D_i^3	D_i^4	D_i^5	D_i^6
1	1	1000	H/2	NA	H	H	0	0	-H
2	1	1000	- H/2	-H	NA	0	-H	-H	0
3	1	1000	- H/ 2	-H	0	NA	-H	-H	0
4	1	1000	H/2	0	H	H	NA	0	H
5	√2	1000	- H/√2	0	-√2H	-√2H	0	NA	-√2H
6	√2	1000	H/√2	√2H	0	0	√2H	√2H	NA

So, here we let us define the terms properly. So, D 0 i, 0 in the superscript i in the subscript it means it stands for the force in member i and that 0 in the superscript means that none of the members has failed. So, it is the intact state of the structure D i j or i in the subscript j in the superscript D j i is the force in member i when member j has failed since we have a structure with one degree of indeterminacy.

So, just one number in the superscript is enough for this problem but in a more general case we would have more numbers in the in the superscript and i and j go from 1 through 6. So, after having defined these terms let us now look at the entire situation. So, we have 6 members and the lengths and areas are all specified. So, let us look at this table column by column. So, the column which is D 0 i so the; force in the structure in all the elements when no member has failed.

So, that is the intact state 0 state and there we have the six member forces negative implies compression but our assumption is that the strength in compression and tension have the same absolute magnitude. So, that is the; intact state and now let us look at the next column. So, when element one has failed and because it is brittle we have just removed it. So, that is D superscript 1 and then with that damage we look at the forces in the remaining member.

So, now we effectively have a determinate structure it is quite simple to solve for member 1 itself obviously it does not have any force because it has been removed for all the other 5 we have three that are loaded and two that are not loaded. So, that is actually going to be useful later on when we try to look at which sequences are going to have what sort of contribution to system failure and likewise we can get D 2 i D 3 i D 4 i D 5 i and D 6 i.

And you see all the diagonal terms are have n a because the member has been removed for that particular column and some of these are not loaded at all in each case we have two elements that are not loaded at all in each case of one member failure and we will come back to this but it is interesting to note that there is some sort of a symmetry in in the loading in between different columns which again will be useful for us when we compute system failure probability.

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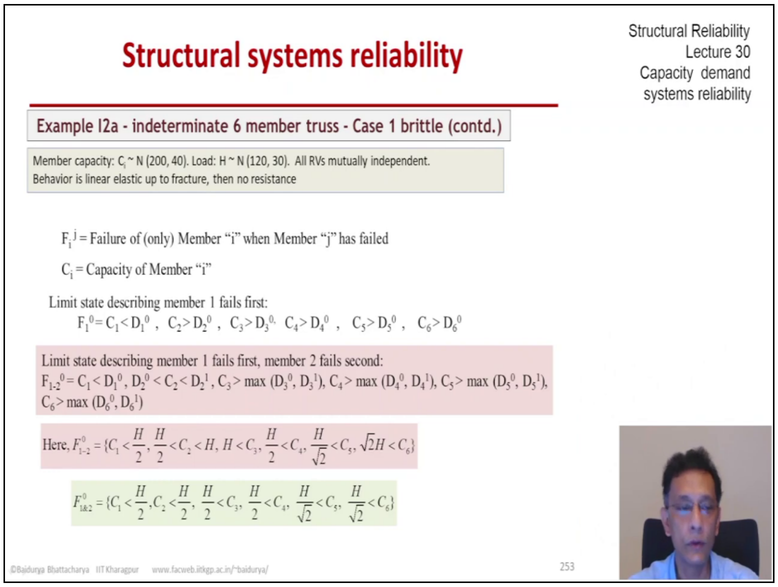

In the next slide let us define failure sequences. So, let us first start with F j i. So, j in the superscript and i in the subscript and that stands for consistent with what we did in the two member parallel system case failure of only member i when member j has failed. And let us also write C i for capacity of member i and let us make the reasonable assumption that the capacity of a member does not change with damage state.

So, even if other members have failed the capacity of any surviving member is the same as it was when the structure was intact now with these two things defined let us define F 01 for example. So, that means that from the intact state member 1 fails first only member 1 has failed. So that would happen if C 1 is less than D 01. So, member 1 is not able to carry the load in intact state but all the other members are able to carry the loads given on them in the intact state. So, that is intersected with C 2 greater than D 02, C 3 greater than D 03 all the way up to C 6 greater than D 06.

So, this then we can take to the next step that let us say member 2 fails second. So, F 01-2 would be the same events that were also that are there in F 01. So, C 1 less than D 01 will be there D 02 is less than C 2 as we had before but then C 2 is no longer able to take the load after member 1 has failed. So, C 2 is less than D 12. So, we have one combined event that C 2 is greater than D 02 but less than D 12 for all the other members they continue to survive the second level loads as well.

So, C 3 is greater than both D 03 and D 13. So, that is why we have the max operator C 4 is greater than max D 04 and D 14 all the way up to c6 greater than max D 06 D 16. So, you are getting the idea that things become quite complicated quite soon when we have this sort of redundant system failure to be modeled. And now putting the values from the previous slide where we had the table of all the loads.

We can write F 01-2 and F 01-3 for all of those different sequences in terms of the C's and H. H is the applied load and we already know what the member forces are under various intact or damaged conditions. Also we could write the simultaneous event F 01 and 2 which is both one and two fail in intact state and the others do not. So, C 1 less than H over 2 C 2 less than H over 2 but all the other ones are greater C 3 greater than H over 2 and so on.

C 6 greater than H over root of 2 so, this way we can define all the necessary sequences if we know which sequences are dominant or and which sequences can be neglected we do not need to do all of them we can only look at a few important ones. But here again we are looking at the entire enumerated set.

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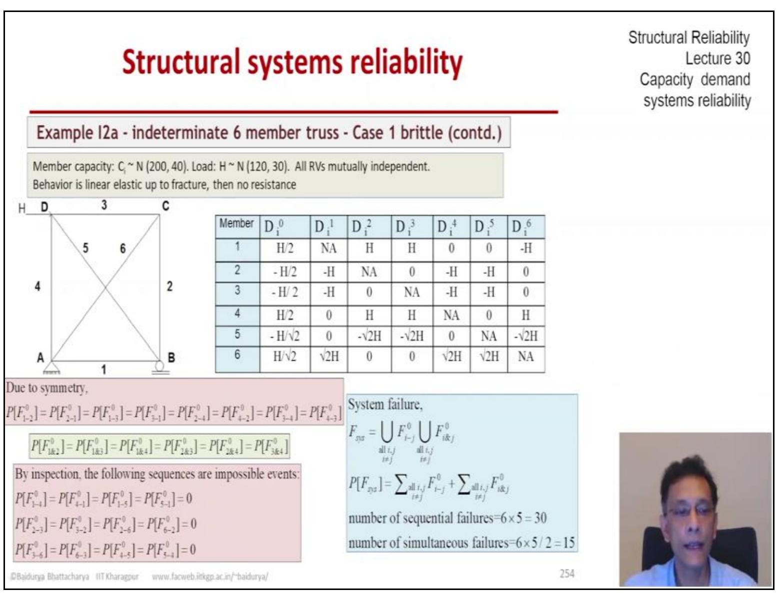

So, this is this is to summarize all these things putting together. So, this is the load table and we can see from the symmetry of the problem that F 01-2 and F 02-1 will have the same probability.

And you see all the other pairs that also have the same probability and that is because the capacities are independent identically distributed and the loads are as they are in the above table. So, this is obviously not a general situation it depends on some of the simplifying assumptions that we have made for this problem and likewise we can see that F 01 and 2, F 01 and 3 etcetera they also have the same probability.

Now we can see by inspection that is because if you see in the load table in the column D 1 i member force and number four and number five have no loads. So, they just cannot fail. So, if member 1 fails first then 4 and 5 have no possibility of failure. So, that sequence 1-4 and 1-5 obviously is not going to happen. So, that is why we see some of those easily identified 0 probability sequences P of F 01-4 F 04-1 all of those have 0 probabilities and it is good to know them because we will not have to worry about them.

We do not have to compute them we do not have to simulate them but this is the description of the entire system failure event it is the union of all the sequences and the union of all the simultaneous failure events and it. So, happens the way we have defined this they are all disjoint because if it is 0 and then 1 then j or i then j then obviously it is going to be disjoint from all other sequences in another event in when another member fails first and some other member failed second.

So, clearly the way we have defined this sequences all are by definition necessarily disjoint. So, it is helpful because we can add the individual probabilities to get the system failure probability there is a P missing after both the sigma signs. So, it is P on the on the right hand side it's P F0 i dash j and P F0 i and j and there are 30 such sequences that we need to consider to be complete and 15 such simultaneous failures that need to be considered. But again as we saw quite a few of them are impossible events.

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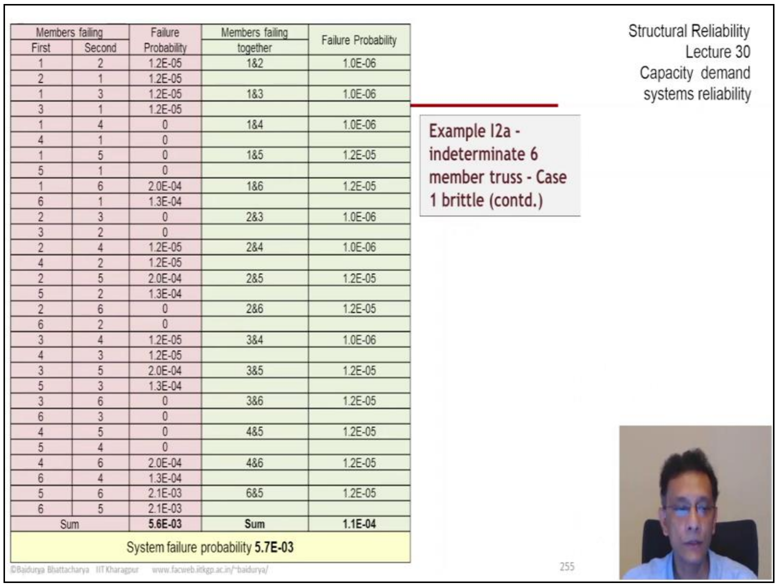

Members failing		Failure Probability	Members failing together	Failure Probability
First	Second			
1	2	1.2E-05	1&2	1.0E-06
2	1	1.2E-05		
1	3	1.2E-05	1&3	1.0E-06
3	1	1.2E-05		
1	4	0	1&4	1.0E-06
4	1	0		
1	5	0	1&5	1.2E-05
5	1	0		
1	6	2.0E-04	1&6	1.2E-05
6	1	1.3E-04		
2	3	0	2&3	1.0E-06
3	2	0		
2	4	1.2E-05	2&4	1.0E-06
4	2	1.2E-05		
2	5	2.0E-04	2&5	1.2E-05
5	2	1.3E-04		
2	6	0	2&6	1.2E-05
6	2	0		
3	4	1.2E-05	3&4	1.0E-06
4	3	1.2E-05		
3	5	2.0E-04	3&5	1.2E-05
5	3	1.3E-04		
3	6	0	3&6	1.2E-05
6	3	0		
4	5	0	4&5	1.2E-05
5	4	0		
4	6	2.0E-04	4&6	1.2E-05
6	4	1.3E-04		
5	6	2.1E-03	6&5	1.2E-05
6	5	2.1E-03		
Sum		**5.6E-03**	**Sum**	**1.1E-04**
System failure probability **5.7E-03**				

So, putting now after the computation we can tabulate all of them and let us look at a few of those. So, let us look at the very first set of rows. So, it is the member sequence one then two and two then one and on the right you have 1 and 2. So, 1-2 and 2-1 they have the same failure probability the failure probability of the one and two event it is almost an order lower. So, in many cases we can actually ignore the one and two type of simultaneous failure events.

Let ss look at the next one 1 3 and 3 1 that was a mistake it should not have happened the system failure priority was supposed to come out last but we will look at this at the end. So, we have 1 3 and 3 1 you see that they are also the same whereas one and three is about an order smaller 1 4 4 and 1 4 4 it is interesting even though the sequences will never happen the simultaneous events do happen.

Te same goes for 1 5 and 5 1 and in fact 1 and 5 is one of the more likely events. So, we will not be able to ignore that here we see for the first time an unsymmetrical situation. So, 1 then 6 and 6 then 1 do not have the same probabilities. So, that is why if we have a cut set we need to find out which ordering of those members in that minimal cut set need to be looked at and others may be ignored and that's here this minimal cut set of 1 and 6 they do not have the same probability in different orders.

So, 1-6 6-1 and 1 and 6 they are all different and this way we can compute all of them and sum them. So, it is 5.6 into 10 to the minus 3 for the sequences 1.1 10 to -4 for the simultaneous events and if you add them the system failure probability is about 5.7 10 to the -3 we need to remember that because the next case we are going to look at is the brittle sorry it is the ductile system and we'll see how the ansys change.

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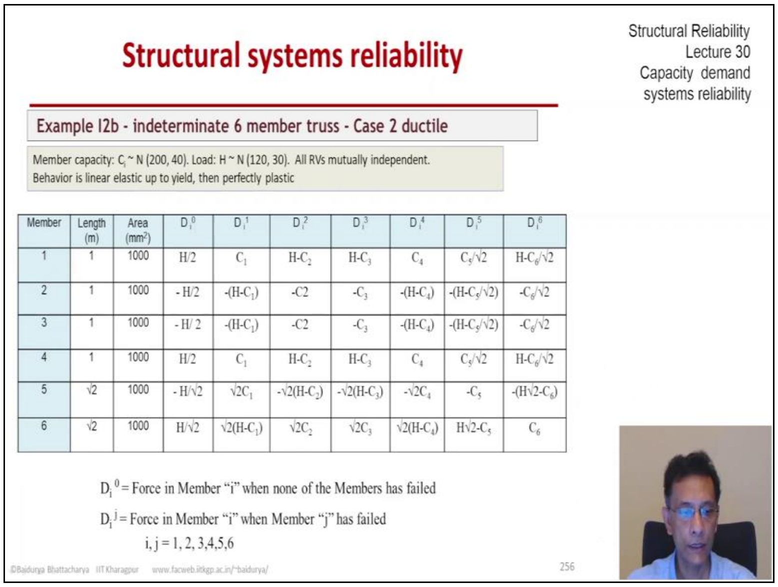

So, we come to example I2b our six-member indeterminate truss with ductile member behavior. So, all the properties are the same as case I2a when the members were brittle in nature. But because of the ductile nature this load table is going to look substantially different. So, let us again recall the definition D 0 i is the force in member i when the truss is intact D j i is the force in member i when member j has failed.

So, the first column is the for the intact structure and it is the same that we had in the brittle situation also but now as members start to fail. So, we have D 1 i D 2 i as in the brittle case we see that it is beginning to look different because now if member 1 has failed it still carries the failure load because it is now fully plastic. So, C 1 is the load present in member 1 effectively and so, that C 1 shows up in all the other member forces and that is what you continue to see when looking at all the other columns.

So, D 2 i D 3 i D 4 i D 5 i and D 6 i, so, all the member failure events now would look a little more complicated uh.

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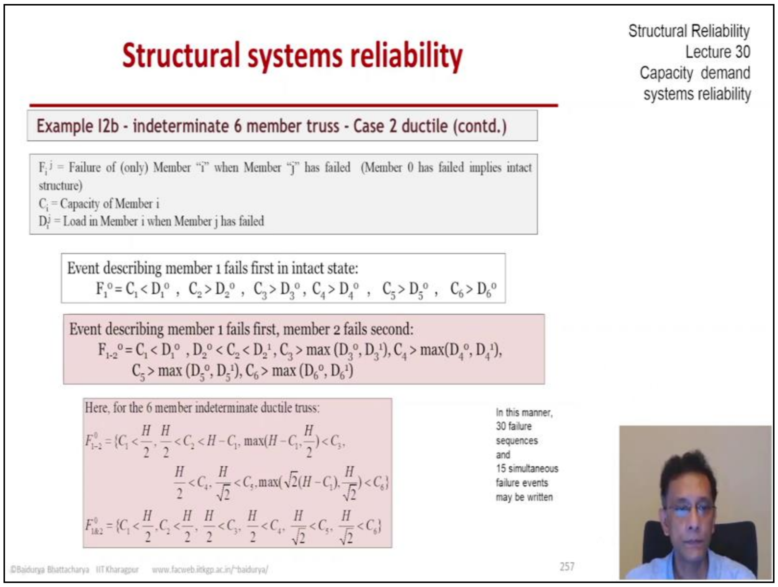

So, let is take a look at that let us define as we did before F j i is failure of only member i when my member j has failed and C i is the capacity of member i as before and D j i is what we just defined. So, F 01 the event describing that member one fails first when the truss is intact. So, again as we did in the brittle case C 1 is less than D 01 and C 2 is greater than D 02 all the way up to C 6 greater than D 06 because we are only interested in number one failing and the others not failing and then we do F 01-2.

So, that is again looking at C 1 has failed and C 2 is between D 02 and D 12 and all the others all the other capacities are greater than the loads put on them at both stages. So, that way we can now bring in the actual load values from the load table in the previous slide and we can define the event F 01-2 where again you see all the max functions because you know we have that that in place already and you see all the C i's there in the in the max also.

So, when one has failed it does not mean that C 1 has vanished. So, C 1 actually shows up in these limit states repeatedly. So, we can do this for all the 30 failure sequences and 15 simultaneous failure events that we described in the previous slide.

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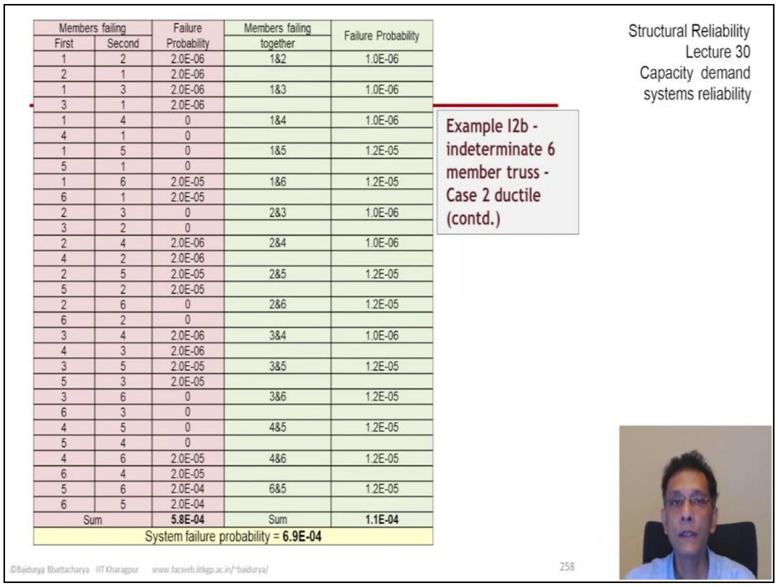

Members failing First	Members failing Second	Failure Probability	Members failing together	Failure Probability
1	2	2.0E-06	1&2	1.0E-06
2	1	2.0E-06		
1	3	2.0E-06	1&3	1.0E-06
3	1	2.0E-06		
1	4	0	1&4	1.0E-06
4	1	0		
1	5	0	1&5	1.2E-05
5	1	0		
1	6	2.0E-05	1&6	1.2E-05
6	1	2.0E-05		
2	3	0	2&3	1.0E-06
3	2	0		
2	4	2.0E-06	2&4	1.0E-06
4	2	2.0E-06		
2	5	2.0E-05	2&5	1.2E-05
5	2	2.0E-05		
2	6	0	2&6	1.2E-05
6	2	0		
3	4	2.0E-06	3&4	1.0E-06
4	3	2.0E-06		
3	5	2.0E-05	3&5	1.2E-05
5	3	2.0E-05		
3	6	0	3&6	1.2E-05
6	3	0		
4	5	0	4&5	1.2E-05
5	4	0		
4	6	2.0E-05	4&6	1.2E-05
6	4	2.0E-05		
5	6	2.0E-04	6&5	1.2E-05
6	5	2.0E-04		
Sum		**5.8E-04**	Sum	**1.1E-04**
System failure probability = **6.9E-04**				

And as we did in the case of the brittle failure we can look at each minimal cut set wise. So, 1, 2 then we have 1, 3 that system failure probability was supposed to come out at last and then 1, 4 and so on. The interesting thing to note is that because of the perfectly plastic behaviour because of perfectly ductile behaviour all the sequences whether it is 1-2 or 2-1 they have the same probability.

And if we just looked at the cut set we would simply be adding 1-2-2-1 and 1-2 to get that minimal cut set probability. So we go through all of them and we sum them we get 5.8 10 to the -4 for the sequential events 1 times 10 to the -4 for the simultaneous events and the sum is about 7 and 10 to -4 compare this with about 5.7 into 10 to the -3 in the brittle case. So, just by replacing the brittle elements with ductile elements the failure probability goes down by a whole order of magnitude.