

Structural Reliability
Prof. Baidurya Bhattacharya
Department of Civil Engineering
Indian Institute of Technology, Kharagpur

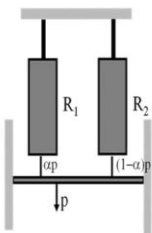
Lecture –222
Capacity Demand Systems Reliability (Part 13)

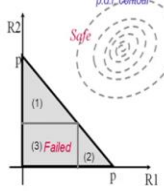
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Structural systems reliability

Structural Reliability
 Lecture 30
 Capacity demand
 systems reliability

• Parallel system (active)






$F_{30} = (1) \cup (2) \cup (3) = F_{12}^0 \cup F_{21}^0 \cup F_{30}^0$
 $F_{12}^0 = \{R_1 < \alpha p, R_2 > (1-\alpha)p, R_1 < p - R_2\}$
 $F_{21}^0 = \{R_2 < (1-\alpha)p, R_1 > \alpha p, R_2 < p - R_1\}$
 $F_{30}^0 = \{R_1 < (1-\alpha)p, R_2 < \alpha p, R_1 + R_2 < p\}$
 Combining, $F_{30} = \{R_1 + R_2 < p\}$
 $P[F_{30}] = P\{R_1 + R_2 < p\}$

p.d.f. contours

Safe

Ductile
 If both are perfectly ductile, it does not matter which fails first



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When we have perfectly ductile behaviour we could take the same approach and define the three regions as we did in the perfectly brittle case but not to worry the mechanics will take care of itself and yield a very different failure region on the R 1 and R 2 space. So, let us start naively that F 6 is the union of 1 2 and 3 which is F 01 - 2 union F 02 - 1 union F 01 and 2 but now let us probe these three events one by one.

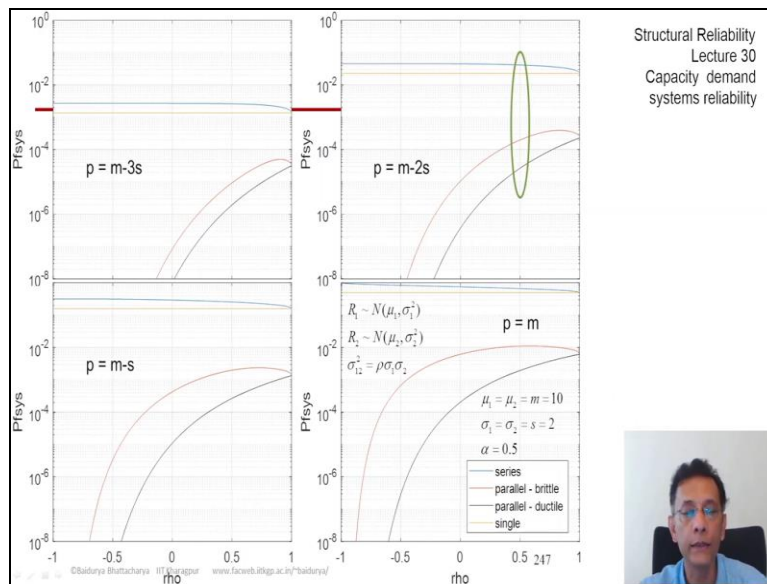
So, F 01-2 is R 1 less than alpha p. So, in intact stage one fails and then R 2 does not fail in the intact stage. So, R 2 is greater than 1 minus alpha P which we have always seen but then the mechanics now makes a difference in the third even that you see R 2 is less than P minus R 1 because R 1 even though it has failed is still carrying that failure load. So, that the third event actually makes all the difference.

And for the second event F 02-1 we get a similar situation R 1 less than P minus R 2 is the new

entrant in the mix and finally in F 01 and 2 R 2 is less than its its load R 1 is less than its load both in the intact stage but the system will fail when whatever their failure loads are together is not able to carry the full applied load. So, that is why you see R 1 plus R 2 less than P as the new event in the mix.

And then if you now combine these three events they are basically all of them saying the same thing that R 1 plus R 2 less than P that would be the final expression for the system failure region and that is why you see the triangular region in the top. And now the system failure probability would be the probability that the sum of the member strengths is less than the applied load. Obviously in a more complicated parallel system we would have a different form but the approach would still be the same.

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Now let us look at some numerical example exactly the same way as we did for the series configuration we have the same sort of strength distribution each normal we have the same set of parameters a mean of 10 standard deviation of 2 and for this particular purpose let us say that the problem is completely symmetric in the two strengths. So, alpha is 0.5. So, the loads are equally shared by the two intact members.

So, with this in mind we have put together 3 values of the applied load the load is still deterministic. So, mean minus three sigma mean minus two sigma mean minus sigma and mean.

So, starting from the top left graph all the way to the bottom right graph that is the corresponding value of P in each case and we have plotted four lines in each of these figures the situation where the two members were in series.

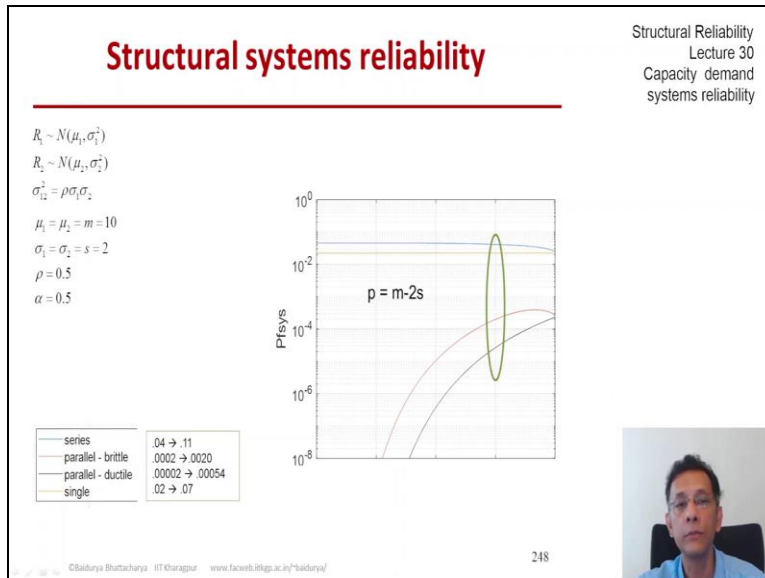
So, that is a throwback to our earlier example the blue line we have also added the yellow line which is what would happen if we had a single element just R_1 carrying the load. So, obviously it would not be affected by ρ , ρ being on the x-axis we have P axis on the y-axis and now the two new lines are the orange line and the black line the orange line corresponds to the behaviour when these two materials the materials of these two members is brittle in nature perfectly brittle and the black line is when the material is ductile in nature.

So, obviously few things we can say immediately is the series system is even worse than the single element system and the series system is way unsafe or less reliable than the parallel configuration in the parallel configuration the ductile is always better and which is obvious because the failure region is smaller. So, the failure the system failure probability will obviously be lesser. But we can see some interesting behaviour with increasing ρ or changing ρ .

While in the series case with increasing ρ the system failure probability decreased for the brittle case we do not see that the brittle case there is actually an optimal value where low is less where ρ is less than one and the system failure probability attains its maximum. So, we don't see a monotonic behavior necessarily in a parallel situation. So, other considerations such as material behaviour would come in the perfectly ductile case that monotonic relationship is still there.

And when the material is perfectly ductile when ρ is one we see the P_f sys the system failure probability is the highest which is exactly the opposite that was in the series system case. Now the last thing we will look at is to consider P to be random and we will fix the mean of P at 6 as we had done in the series case and let it be a gumball random variable with COV 30%. Exactly as we had before and let us see how the numbers change.

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So, the green oval that I had marked in the previous slide and what you see here on the right is how the numbers change when P is a constant equals 6 and P is a random variable with the mean equals 6. So, for the series system we have already seen the answer it goes up from 0.04 to 0.11 the system failure probability for the single it goes up from 0.22 to about 0.07 and the parallel brittle situation goes up by almost ten times 0.002 and for the parallel ductile system also by a little more than one order 0.002 to 0.00045.

So, that brings an end to this comparison between two unit series and parallel system and the two extreme cases of parallel behaviour brittle and dark time.