Structural Reliability
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Lecture –221
Capacity Demand Systems Reliability (Part 12)

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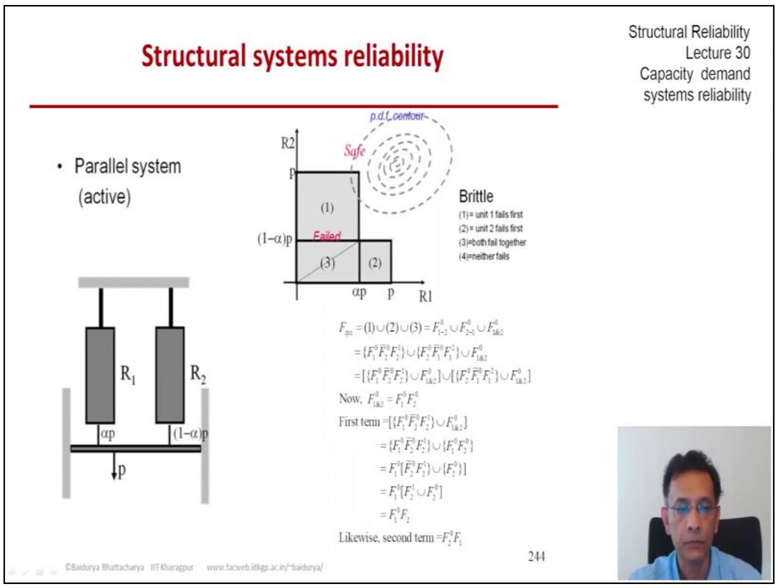

In the perfectly brittle case we can describe the failure region as the union of three disjoint regions and these three regions are actually quite revealing and it might aid the analyst for a much more complicated situation to proceed systematically through a sequence of member failures. So, let us look at them one by one. The top block marked one gives the situation where unit one fails first followed by unit two. So, R 1 is not able to carry the load alpha P.

So, R 1 is less than alpha P and then because it is brittle the entire load goes over to R 2 the second member and it is also not able to carry that load. So, R 2 is less than P. So, together they define the region marked one likewise the region marked two is when unit 2 fails first and that has a similar logic to it. The region mark 3 is interesting it is that situation where one and two fail simultaneously in some sense.

And so, there is no scope of redistribution we each member is weak enough. So, that it is not able to carry the load given to it in the intact stage now let us define the symbols on the right part of the equation the right hand side F 0 1-2 means failure starting from intact condition 0 means no element has failed. So, that is the intact state and then in the subscript we have 1 dash 2. So, one fails first followed by two union we have F 0 in the superscript 2 dash 1 in the subscript. So, it is the same logic starts from intact case followed by failure of two and then failure of one.

And the last one is F 0 subscript 1 dash 2 as I said that is both of them fail together and when the system is in intact state and we are going to expand that in terms of two events very soon. So, let's expand F0 1-2 and see what events actually it is composed of and this approach is going to help a systematic structural analysis of an increasingly damaged system. So, let us look at the three events in the first set of curly brackets. So, it is F 01 F 02 bar meaning the complement and F 12.

So, let us see what they are trying to say F 01 means in the intact stage 1 member 1 fails that is fine but to be able to claim that unit 1 fails first we also have to specify that unit 2 does not fail first. So, that is where the F 02 complement comes in use. So, F 02 complement tells us that in the intact state 2 is not failing and then once one has failed the third event gives the next condition that in the state that one has failed in that damage state 2 is the next member to fail.

Likewise the second set of events inside the curly brackets gives the region two which is you know two fails first and the third even we have. So, far kept intact F 01 and 2 kept unchanged but we are going to expand that also in a few seconds. So, what we do next is we have three events A union B union C and we just for convenience or for a certain purpose we are going to do A union C or B union C.

So, we bring F 01 and 2 along with the first set of curly brackets and now it is logical to say that F 01 and 2 is simply the event the intersection of F 01 and F 02 why because all we are saying is that in the intact state one fails and in the intact state two phase. So, that is why there is no scope of redistribution no analysis of a partially damaged structure F 01 and 2. So, with that explanation let us look at the first term within the square brackets and that is F 01 and F 02 bar and F 12 the whole thing union with F 01 and 2.

And now bringing in the expression for F 01 and 2 we find that F 01 is the common event in the two sets that have the union. So, we can take that common out and express that as the intersection of F 01 and another 2 sets and then we also see that in those events in the inside the square brackets we have F 02 complement and F 02 the so, the same event and its complement.

So, it further simplifies to the union of F 12 and F 02 now that even that union saying that F 2 fails that two fails after one or two fails in intact stage it basically means two fails that is all. So, if we interpret that and give a symbol that it is f subscript two then we can define the first set of events as F 01 intersection F 2. Likewise the second term becomes F 02 intersection F 1. So, our system failure event could also be expressed as the union of 3 events not union of 3 events.

And obviously you sense the difference when is the union of 3 events as we have in the top it is one or two or three these are all disjoint sets. So, in some sense it might be easier to analyze them the probability of the union of disjoint sets is simply the sum of the probabilities. So, it might be easier on the other hand F 02 and F 1 or F 01 and F 2 these events might be easier to conceptualize and to analyze a damaged structure. So, in some sense even though they are not disjoint it might be easier from a structural analysis point of view to define failure that way.

So, we are not giving any preference at this stage we are just giving both options which can be used by the analyst depending on the situation.

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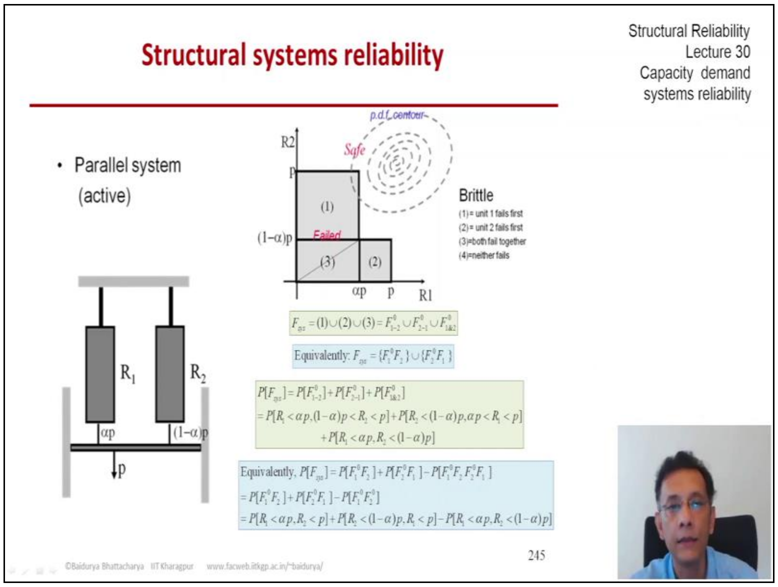

So, now we are ready to define the system failure as I said when we have the union of three disjoint sets the failure the probability of the union is just the sum of the individual probabilities. So, that is what you see and now we are able to bring in mechanics and define the three disjoint sets in terms of the loads and the member strengths. So, just going through the steps F 01-2 is the same as R 1 less than alpha P and R 2 between 1 minus alpha P and P.

Likewise you get the second term and the third term which is F 01 and 2 is R 1 less than alpha P and R 2 less than 1 minus alpha P. So, together these three are terms can be computed and we can find the system failure probability as I said we could also equivalently define system failure as the union of two intersecting sets and in the end we are going to get the same answer. So, we express F sys as P of F 01 F 2 + s p of F 0 to f 1 minus the probability of intersection and if you go through the steps carefully you end up with the sum of two probabilities.

And a subtraction of the intersecting area but in the end they will give you the exact same answer again depending on which we one do we choose depends on the particular structure and how easy it is to analyze one way or the other.