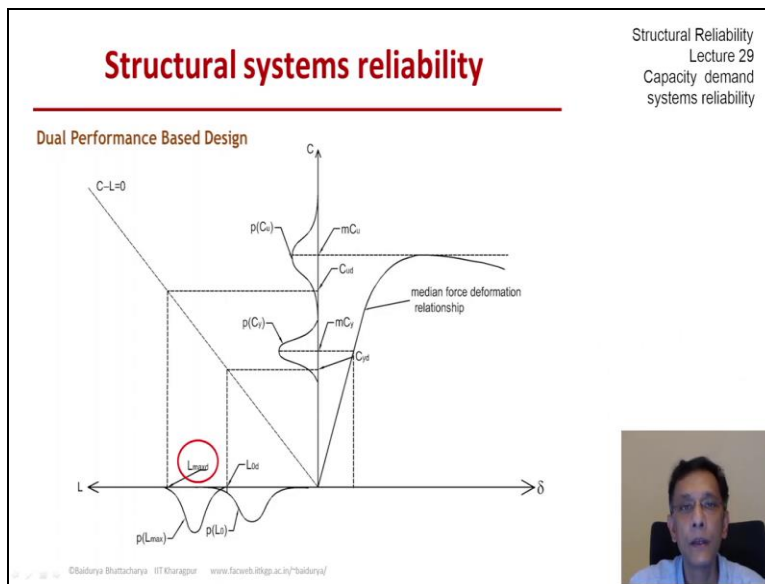


Structural Reliability
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Lecture –218
Capacity Demand Systems Reliability (Part 09)

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We encounter series systems when considering multiple performance levels and multiple load combinations for a structure. Let us take up the multiple performance levels first and here is one such schematic for a dual performance level situation. So, this diagram is a bit detailed. So, let us go through this part by part. On the right part you see the C delta relationship. So, we can think of delta as being a response and C is some measure of capacity.

So, it is a non-linear relationship and what we have plotted here is the median curve between C and delta which means that if we ran this process again and again we would get several stochastic processes and what you see there is the median line. So, from this median line we could obtain the yield capacity corresponding to some displacement delta predetermined we could get the yield capacity which we call let us say C_y and it would have its mean or median that is indicated in terms of mC_y.

We have the density function which is indicated as p_{C_y} and other metrics if you like m_{C_y} is the median we could also define the design value C_{yd} typically less than mean sort of conservative value. So, we could define those quantities for yield. We could also define the ultimate capacity also from this force deformation type relationship and that is what you see on the top marked in the green circle now and likewise as was in the yield we can define the density function of C_u and the median ultimate capacity m_{C_u} the design ultimate capacity and so on.

So, now with that in mind let us let us look on the left side of the figure. So, if we have two levels of capacities for the same structure it is quite reasonable to suppose that we would consider those as acceptable limits under different intensities of loads. So, for moderate loads we would be more interested in yield and or the structure does not yield and for high loads extreme loads we might be more reasonably interested in the structure does not collapse it does not fail. So, this density of the yield capacity C_y should it seems be compared with an ordinary type of load.

So, that is what you see on the left marked in green circle is the density of the ordinary load L being the load that is the L axis and L_0 is the ordinary load PL_0 is the density function and there you can define its design value and compare it with the design capacity and make sure that the two match and thereby the design can be satisfied. Likewise the ultimate capacity is concerned is relevant when we have extreme loads.

So, on the L axis we can define L_{max} and its density and its design value and compare with the design capacity and satisfy a design. So, this gives an idea about for the same structure we can be interested in two or more performance levels set different requirements which would also be governed by different regimes of structural mechanics maybe for yield or below yield we will be happy with linear analysis elastic analysis and for near ultimate we would really need to consider beyond yield behaviour plasticity non-linearities and so on.

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Dual Performance Based Design

FEMA 350		
	Performance Levels	
	Immediate Occupancy	Collapse Prevention
Demand level	500 yr return period earthquake	2500 yr return period earthquake
Non structural requirements	Equipment and contents should be OK, may not work due to lack of power	Extensive damage allowed
Structural requirements	Strength and stiffness must be retained. Minor cracking allowed. Elevator and fire protection systems must be OK.	Little strength and stiffness remains. Gravity loads must be supported. Large permanent deformation allowed.



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So, and we see this sort of thinking in modern design codes for example this is an extract from fema 350 and you see that there are there are two limit states or performance levels the immediate occupancy and the collapse prevention. And each of these has a very different demand level different set of requirements both for non-structural and structural parts. So, for example the demand for immediate occupancy is a 500 year return period earthquake.

Whereas for collapse the demand level is much higher at 2500 year return period and you can read the other aspects also and it is clear that the performance levels are different the expectations are different the demands are different and presumably the acceptable safety should also be different.


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Multiple Performance Based Design

Performance Level	Limit State
Immediate Occupancy (FEMA-273)	$g = 0.007 - IDR_{max}$
Life Safety (FEMA-273)	$g = 0.025 - IDR_{max}$
Collapse Prevention (FEMA-273)	$g = 0.05 - IDR_{max}$



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We can look again from fema 273 there is another example there are three levels. Now instead of two there is immediate occupancy there is life safety and then collapse prevention and each of them have different limits for the maximum interest stored drift ratio IDR and you see how they increase with more intense and more severe limit states, so, from 0.007 to 0.025 to 0.05. So, with these in mind we can define a very small example.

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Example: dual performance limit states

A structure, with resistance R , is acted upon by dead and live loads. Two loading conditions are relevant:


- (i) Dead load (D) + Maximum live load (L_m) with structural capacity C_u
- (ii) Dead load (D) + Ordinary live load (L_o) with structural capacity C_y

The structure must not collapse under (i) and must not yield under (ii).

The mean and c.o.v. of each random variable are given in parentheses:
 C_u (100 kN, 20%), C_y (60 kN, 10%), D (20 kN, 10%), L_m (40 kN, 40%), L_o (20 kN, 20%).

Assume the random variables are independent of each other, and each is Normally distributed.

- (a) Find the reliability index for each load condition.
- (b) Find the correlation between the two limit states.



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And let us see if we can learn something from this. So, let us say we have a structure which is acted upon by two sets of loads dead and maximum live load and the second being dead and ordinary life load. And under the maximum live load situation the structural capacity which is relevant is C_u and for the ordinary live load we have C_y and the requirement is that the

structure must not collapse under one meaning the load combination or the demand one and it must not yield under two.

So, two different sets of expectations we have here the all the distribution information of the four random variables C u C y 5 random variables C u, C y, D, L m and L o, o being ordinary and m being max. So, under the assumption that all the random variables are mutually independent and just for the sake of simplicity which is normally distributed we can find out what the reliability index is for each limit state. And it is an interesting exercise is there any dependence any correlation between the two limit states between the two safety margins.

So, if you would like to solve this problem yourself please pause the video or I am going to present the answer in the next slide.

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Example: dual performance limit states (contd.)

$$M_u = C_u - D - L_o, M_u \sim N(\mu_u, \sigma_u^2)$$

$$M_y = C_y - D - L_o, M_y \sim N(\mu_y, \sigma_y^2)$$

$$\mu_u = 100 - 20 - 40 = 60$$

$$\sigma_u^2 = 20^2 + 2^2 + 16^2 = 660 = 25.7^2$$

$$\mu_y = 60 - 20 - 20 = 20$$


$$\sigma_y^2 = 6^2 + 2^2 + 4^2 = 56 = 7.48^2$$

$$\text{cov}(M_u, M_y) = E(M_u M_y) - E(M_u)E(M_y) = \sigma_D^2$$

$$\rho(M_u, M_y) = \frac{\text{cov}(M_u, M_y)}{\sigma_u \sigma_y} = \frac{4}{25.7 \times 7.48} = 0.021$$

$$\beta_u = \frac{\mu_u}{\sigma_u} = 2.33$$

$$\beta_y = \frac{\mu_y}{\sigma_y} = 2.67$$



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So, here they are we define two safety margins m u and m y and you see how they are just the linear combinations and each is normal with their own mean and variance. And with the given numbers we can find that the mean of mean is 60 and variance is 25.7 squared and the mean of m y is 20 and the variance is 7.48 squared. These I hope my numbers are correct and now with this we want one question was what the correlation coefficient between the two limit states is.

So, we find the covariance and divide that by the two standard deviations and the answer is

almost zero it's about 2.1%. So, because D is the only common random variable there if there was dependence between the C_u and C_y there would have to be a strong case for that but then we would probably see higher correlation coefficient between the two safety margins. And finally the beta values for these two limit states which are mean divided by the standard deviation it's 2.33 and 2.67 in ultimate and yield respectively.

Now whether these are adequate whether we should have a higher safety margin in ultimate obviously is a very valid question and we will like to take this up towards the very end of this course in part D.

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Example: Two load combinations

A structure, with resistance R, is acted upon by dead, live and wind loads. Two load combinations are relevant:

- (i) Dead load (D) + Maximum live load (L_m)
- (ii) Dead load (D) + Ordinary live load (L_o) + Maximum wind (W).

The structure fails if either of the load combinations exceeds the resistance.

The mean and c.o.v. of each random variable are given in parentheses:

R (100kN, 10%), D (20 kN, 10%), L_m (40, 40%), L_o (20kN, 20%) and W (20kN, 40%). Assume the random variables are independent of each other, and each is Normally distributed.


- (a) Find the reliability in each load combinations.
- (b) Find the correlation coefficient between the two safety margins.

Answer:

$$M_1 \sim N(40, 19^2) \Rightarrow \beta_1 = 2.1$$

$$M_2 \sim N(40, 13.6^2) \Rightarrow \beta_2 = 2.9$$

$$\rho(M_1, M_2) = 0.40$$



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Let us move on to the next related problem under this topic is two load combinations. So, we discussed two performance levels now let us have two load combinations and let us say the resistance is common in both these two load combinations. So, we have dead plus maximum live load and the second load combination is dead plus ordinary life load plus maximum weight and the structure has to survive both.

Here you see the all the random variables again there are 5 of them and are being common. So, one could expect that even though all these 5 are mutually independent that there would be a good amount of correlation between the two safety margins. So, the first question is we need to find the reliability in each of these two load combinations and the question b is find the

correlation coefficient.

And as before we take the five random variables to be normal with the mean and COV as given and each are they are all mutually independent. There is one typographical error the l_m has a mean of 40 kilonewton not just 40. Anyway so if you want to work this walk through this then please pause the video otherwise let me present the answer which is this m_1 the first safety margin is a normal with mean 40 and standard deviation 19.

So, that gives a beta of 2.1 and m_2 is normal with mean 40 and a standard deviation of 13.6 and beta2 is 2.9 and the correlation coefficient between these two safety margins is 0.4.