Structural Reliability
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Lecture –217
Capacity Demand Systems Reliability (Part 08)

We started our discussion on series system reliability with the simplest possible configuration of a series system namely two prismatic bars connected in series axially loaded by the same force P. In that context we looked at the effect of dependence among the two member strains and also uncertainty in the load. Let us continue with that line of enquiry but let us bring in more elements higher than two.

And let us give the option of not every member being loaded by this same force at the same level. So, a 2D determinant truss gives us just the right platform for that.

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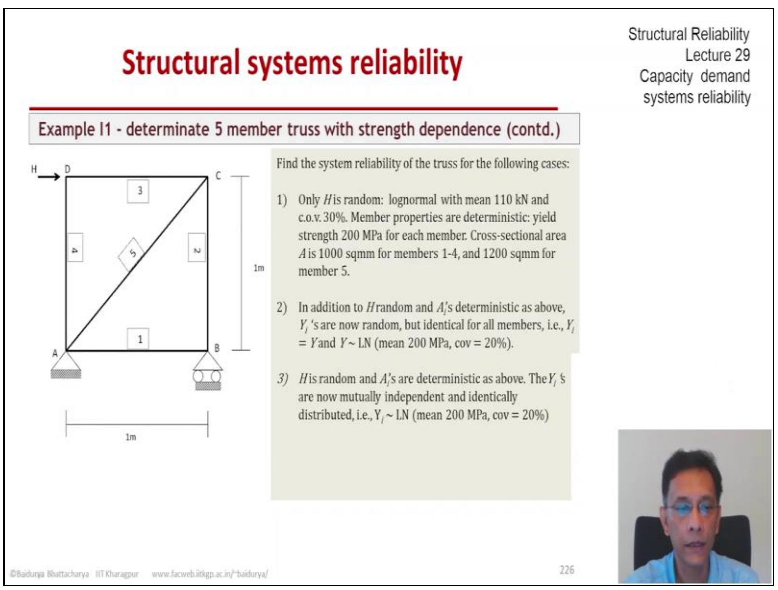

So, we look at a five-member truss a square truss with one diagonal and we will consider a few cases. So, here are the scope of the problem and all the random variables. So, we have one horizontal force each element can fail either in compression or in tension failure is defined in terms of stress the stress magnitude in failure whether in tension or compression the magnitude is the same and the behavior is linear elastic.

So, it is as simple as things can get the element limit state is E times area minus the member force and the system limit state the system failure is the union of these element failures. So, as I said we will look at four different cases the first case is only the load is random not normal with a certain mean and COV and in this case all the member properties including yield strength are deterministic. All the four sides of the square have the same area and the diagonal is a little thicker.

In case 2 we introduce randomness in the member strengths the Y's all the yield strengths are all random but they are the same. So, they are identical and fully dependent H continues to be random as before and the cross-sectional area they are as defined before and non-random. In case three we go to the other extreme for the member strengths instead of being fully dependent. Now they are fully independent. So, they are mutually independent and identically distributed.

So, here we have 1 + 5, so 6 random variables and in the final case we go to the midway. So, H is random the A's are deterministic but the member strengths are partially dependent. So, they have the same correlation coefficient between each pair of 0.5. So, let us look at what happens in each of these four cases and let us also set up and solve the problem.

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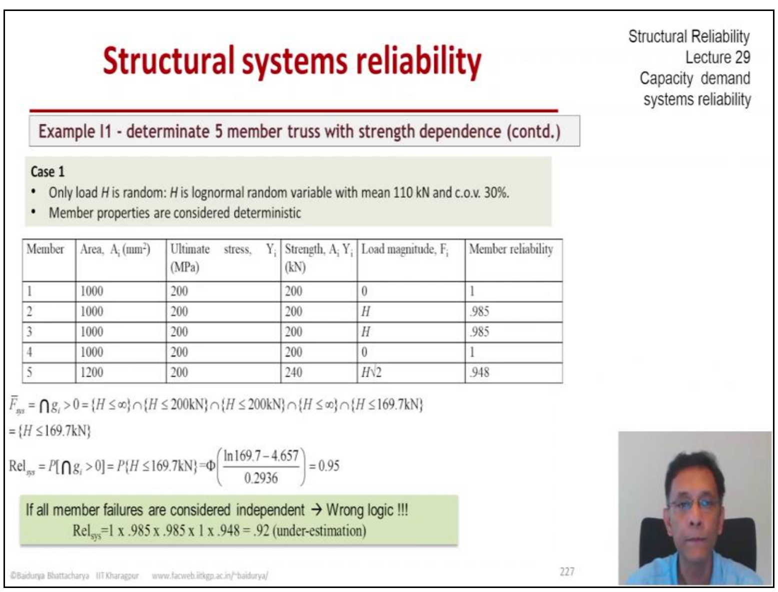

So, in case one a very simple structural analysis gives us all the member forces. So we have in the column on the right but one the load magnitudes and if we compare that with the member strength which is area times yield we can find the probability that the strength will be greater than the member load F i and on the very right column that is why you see we have listed the member reliability.

In member 1 there is no force that is a consequence of the kind of loading on the structure in member 4 there is also no load. So, those two members have full reliability there is no possibility of failure and for the other three members, members 2, 3 and 5. The member reliabilities are as you see on the right column 0.985, 0.985 and 0.948. It would be tempting as is done in cases of a series system with independent elements to multiply all these reliabilities and come up with a system reliability that would not be right.

Because in structures we are sharing the same load between the elements we have to be cognizant of that. This is the system performance event the system is safe the complement of the system failure and that is all the member limit states are positive. And that written in terms of H is given as the intersection of those 5 events as you see in the first line of that equation H less than infinity is basically saying that Y has to be greater than zero.

So, that is obvious there is no possibility of failure in members one or four and for the others you see H has to be less than 200 kilonewton for member 2 h again has to be less than 200 kilonewton for member 3 and less than 169.7 kilonewton for member 5. And now since we are looking at the intersection of these events for the system safe performance the intersection is H is less than 169. So, it in effect becomes a single element problem once we take into account the repeated appearance of H in all these limit states.

And now we are able to introduce the log normal nature of H we have been doing this sort of thing in elementary liability computations. So, we know that quite well and that can be given the answer can be given in terms of the normal CDF evaluated at log of 169.7 minus the logarithmic mean divided by the logarithmic standard deviation and the answer is 0.95. Obviously if we had naively multiplied the member reliabilities we would get a pessimistic answer it would be obviously a wrong logic and the system reliability would end up being 0.92 in that case but that would be the wrong answer.

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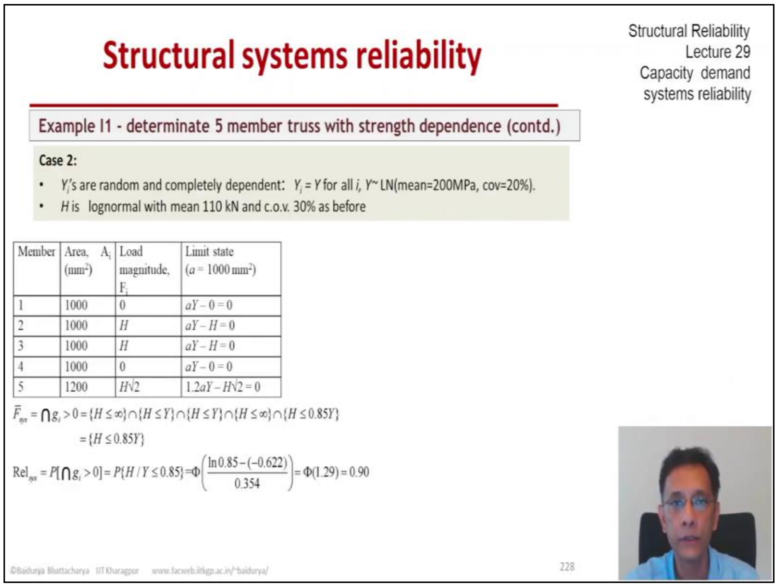

We move to case two next and case two is we allow all the yield strengths to be random but it is the same yield strength for all of them. So, all the 5 strengths are completely dependent here in this table we have listed the limit states and we again look at the system safety event. So, that given in terms of the H and Y's, H being the first member and the fourth member there is no failure obviously. So, looking at members two three and five the most stringent condition is H is less than 0.85 Y.

The reason we can say that is that both H and Y are non-negative quantities. So, H less than Y and H less than Y and H less than 0.85 Y the answer is H the intersection is H less than equal to 0.85 Y. So, again H and Y being log normal random variables we can convert that limit state into a single limit involving a normal random variable and that is what you see in the next line. We again come down to a single normal CDF the beta value is 1.29 and the reliability is 0.90.

So, from the earlier value of 0.95 we come down to 0.90 and in the end we are going to look at all the four answers four cases together and see how they all fit together.

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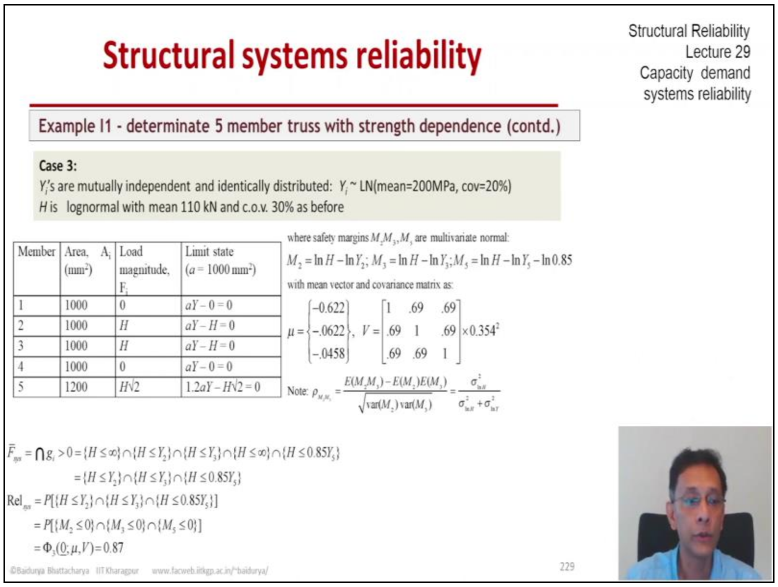

Let us move on to case number three which is all the yield strengths are now mutually independent. So, in case two they were all the same now they are all mutually independent and that unfortunately we are no longer able to convert the system safe performance event into one single limit state but we have to now take into account the fact that all the Y's are different. So, Y 2 and Y 3 and Y 5 are different random variables and the three events are also not independent because H is common to all of them.

So, we have to be careful of that fact but it is also true that H and Y they are all log normal. So, we can guess that we are now looking at three dependent normal safety margins. So, that gives us three safety margins in terms of m 1 sorry in terms of m 2, m 3 and m 5 m 2 being defined as log of h minus log of y 2 because we want to go to the normal space and likewise for m 3 and m 5 and that gives that lets us compute the mean vector of the m's.

And the covariance matrix of the m's the way we do that is if you want to work out the long hand the covariance or the correlation coefficient can be computed as you see on the screen and then we are able to go back to the joint CDF of m 2 less than m 2 at 0 m 3 at 0 and m 5 at 0. So, that intersection of the three of the three events has the probability of 0.87 and we compute that using the multivariate normal function which we know how to do in MATLAB. So, it further comes down to 0.87.

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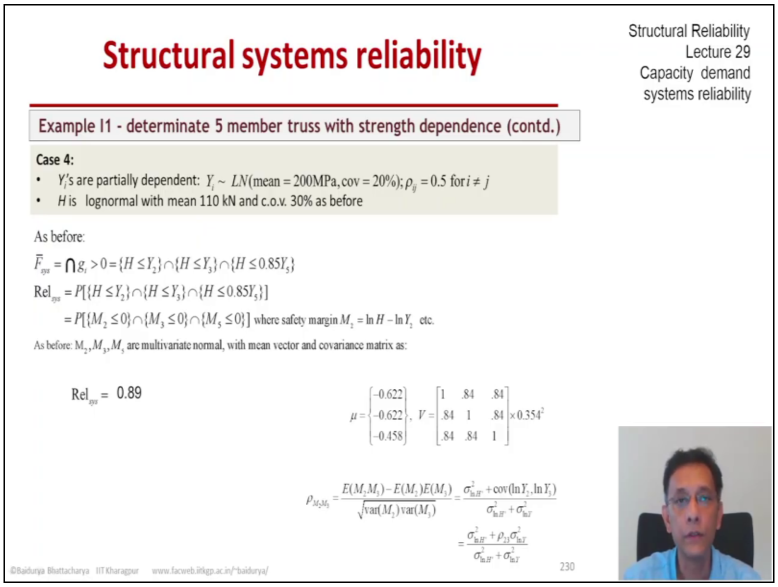

In the fourth case we allow some dependence in the member strings to be specific rho of 0.5 between each pair. So, we proceed as before we have three we have three safety margins m 2 less than equal to 0 m 3 less than equal to 0 and m 3 sorry m 5 less than equal to 0 and we want to look at the intersection of those three events. And we are able to find the mean vector and the covariance matrix while computing covariance we have to also take into account the dependence between the Y's.

So, Y 2, Y 3, Y 3, Y 5, Y 2,Y 5 and etcetera. So, we do that and we have we have the results here the mean vector of the m's and the covariance matrix of the m's are as you see on the screen if you wanted to know how to do that i have given one example for m 2 and m 3 where you see that rho of 23. So, which means row between the Y 2 and Y 3 also need to be listed in this case and finally the answer comes to 0.89. So, it goes up from the complete independence case.

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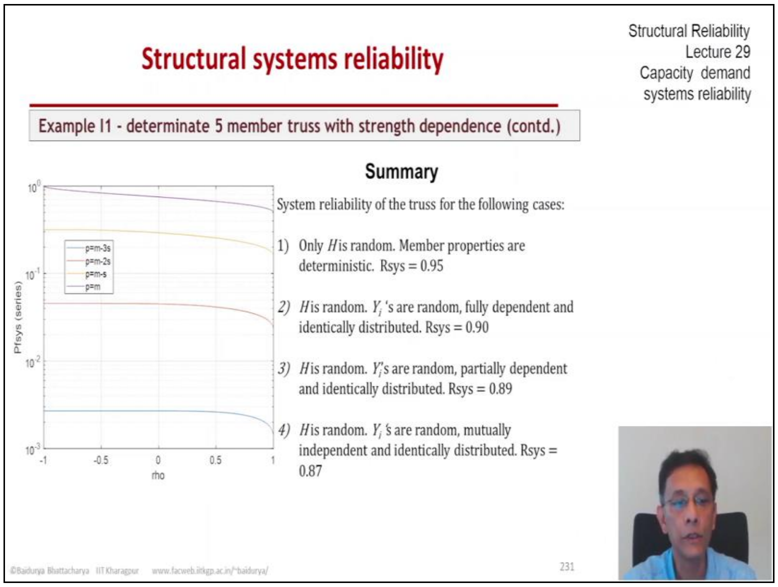

So, putting together all these four cases we find that when H is the only random quantity the member strains are non-random then we have the highest R sys and then when we allow randomness in the member strengths but consider them to be fully dependent then it comes down from 0.95 to 0.90 when we allow partial dependence in the member strings. Then it further goes down to 0.89 and when they are fully independent all the member strengths that is then we have the lowest system reliability of 0.87.

And this is actually evocative of what we have already discussed in the beginning of these discussion on series reliability looking at the two element two bar system that we looked at here on this graph you see a rho on the x-axis and as we saw in all those four cases of different load magnitudes that as dependence increases system reliability goes up system P f sys goes down. So, reliability goes up and as we go towards independence P f sys goes up which means reliability goes down and this is exactly what we saw in the case of this truss as well.