Structural Reliability
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Lecture –216
Capacity Demand Systems Reliability (Part 07)

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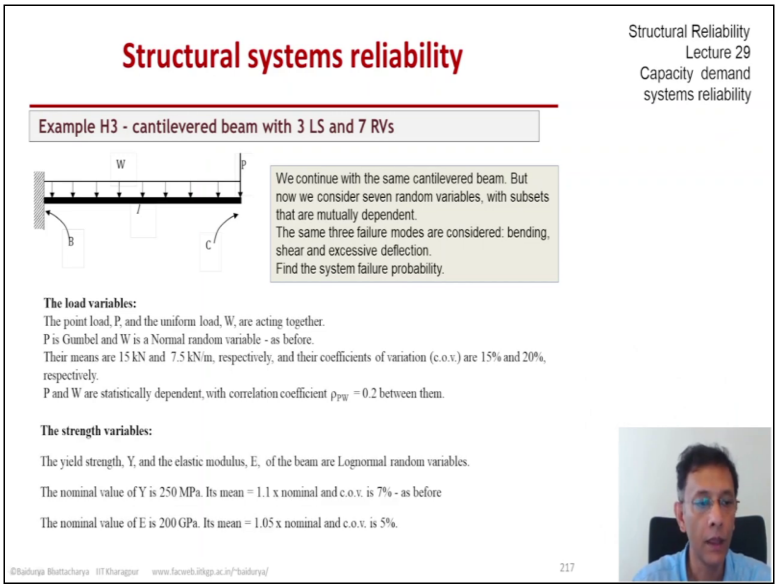

We continue with the same structural member for this example but now we consider 7 random variables in place of 2 as we did before. And we are going to solve these using Monte Carlo simulations as you see there is dependence among these seven random variables which we are going to describe shortly. So, let us go through them step by step we have the same 3 limit states as we did before.

Some of the numbers some of the descriptions will be different from problem h 1 and we are going to use the SI system for units. So, the load variables are P and W P is gumbel W is normal however their means are given in terms of kilonewton and clinton per meter and they are dependent. So, we have partial information about their dependence given in terms of their correlation coefficient which is 0.2.

The strength variables there are 2 of them Y and E the yield strength and the Young's modulus.

Now they are both random variables they are both log normal and they are mutually independent. Because there is no reason to have yield and stiffness have any dependence between them we do not observe that usually.

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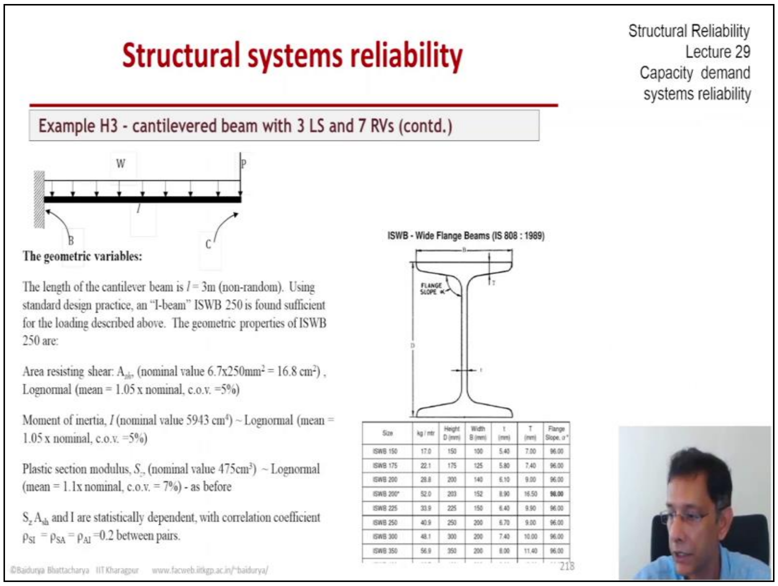

The geometric variables the span is now 3 meters which is non-random now let us now use some real sections and values of these geometric properties through a design. So, let us say we have gone through the design and it turns out that ISWB white flange beam 250 is is the appropriate beam selected for this load and the geometric properties along with the randomness is R. So, the area resisting shear has a nominal value of 16.8 square centimeters.

So, let us say let us a log normal random variable with mean 1.05 times the nominal which means the bias factor is 1.05 and a COV of only 5%. Likewise the moment of inertia which is almost 6000 centimeter to the power 4 is also considered to be log normal which is a common assumption with again a bias of 1.05 and a small COV of 5%. The plastic section modulus is has a nominal value of 475 cubic centimeters and the bias is 1.1 here and the COV is a little larger at 7%.

So, these define the geometric random variables but now they are mutually dependent. So, again we have partial information about their dependence it is the correlation coefficient between each pair is 0.2 and then the failure modes.

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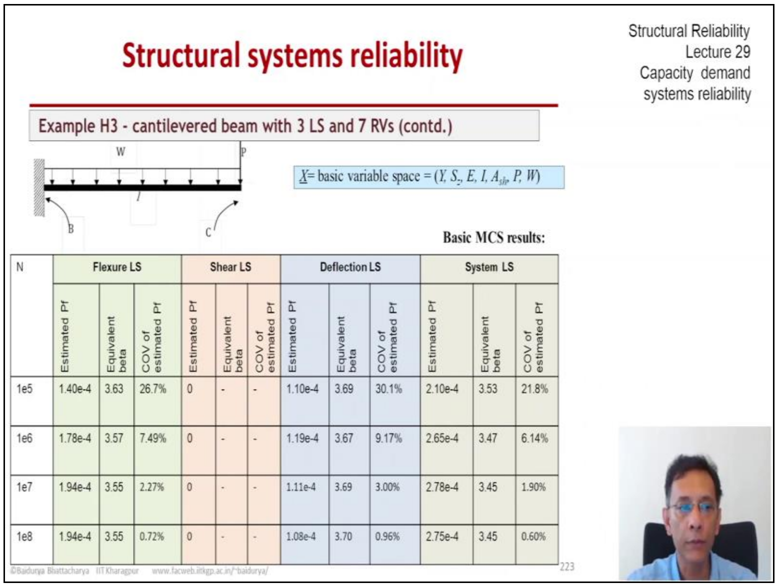

So, after defining these seven random variables the failure modes there are 3 of them as we have already discussed in the previous example flexural failure shear failure and deflection failure and these are the forms of the limit state equations you have the applied load of the applied moment and applied and the moment capacity. So, let us a capacity demand type formulation. So, we have for bending and we have for shear and we have for deflection this time the limit on reflection is span over 100.

So, putting all of this together now we could actually write our Monte Carlo code and go through and take it to solution. So, let us go through the steps of the MATLAB program that I have written for this purpose we first define all the random variable parameters. So, we define the mean of P the COV of P and because let us gumballs you obtain the corresponding u and alpha values we define the mean and COV of W which is normal.

Then we define the correlation matrix between P and W and again to save space I have written it out in one line. So, rho of PW is 0.2 and the correlation matrix is 1 rho PW semicolon rho PW 1. So, that is a 2 by 2 matrix and then because I will need this during simulations the Cholesky factor of the correlation matrix. I would like to have the lower triangular form.

So, that is why I take the transpose of that because somehow MATLAB gives the upper triangular form. And then because I am going to use these later I initialize the vector Y for PW 1 and 2. And I am going to need that because I am going to combine 2 independent normals through this sort of Cholesky factor. Then I go to the geometric values. So, I am sorry I go to the strength values. So, the yield strength and the Young's modulus Y and E they are independent and I obtain their corresponding log normal parameters.

And we need to remember that they are in different units. So, one is in Megapascal, one is in gigapascal. So, once we put all of them together in the Monte Carlo loop we need to make sure that we are respectful of the units. Then we look at the geometric quantities. So, I have the I the section moment of inertia which is again log normal. So, I obtain those parameters S z which is also not normal. So, I obtain their parameters and the area shear which again log normal and i obtain its parameters.

So, I have defined them and it so, happens they are all in terms of centimeters centimeter power 4, 3 and 2 respectively. But then I need to incorporate their dependence. So, I have been given the correlation coefficient between each pair. So, and which happens to be 0.2 for each of them. So, I need to define the 3 by 3 correlation matrix which you see in the first line there let us one rho SA then next line rho SI one rho AI and the last line is rho SA rho AI and A. So, these are the the 3 by 3 coefficient matrix between S z, I and inertia.

And like I did for the loads I need the lower Cholesky factor which is what you see in the second line and I initialize the vector that I will need for the Monte Carlo simulation. Next and yes the length is 3 which is non-random and let us in meters. So, we have to be mindful about all these different units. Next we are ready to start the Monte Carlo loop. So, I initialize the counters.

The count any failure count flexure for flexural failure count shear for shear failure and count deflection for deflection failure and I am going to need some of these moments of the estimate. So, I initialize some I flexor and some square I flexor as well and I also need to define the number of multiple trials here I seem to have about 100 million simulations. Let us I think the last case that I studied.

Now we enter the Monte Carlo loop I generate the independent Y's and E's. So, that is that simple and I need to make sure that I am being consistent with the units. So, that is what you see in these 2 lines rand n being the standard normal random deviate then I need to generate the dependent set i and a shear and for that I generate 3 independent normals zz 1 zz 2 and zz 3 and then I obtain the intermediate Y using the linear trans combination of the zzz. So, that is where that lower Cholesky factor comes in and then from each of the Y's I obtain Azi and a share through the normal transformation.

So, that gives me the 3 quantities and there I have also been mindful of the units then the loads are generated in a similar manner I first generate 2 independent standard normals z 1 and z 2 and then combine them with the lower Cholesky factor and then transform each of the Y's to P and W respectively I am again mindful of the units then I introduce 3 flags because I want to keep track of how many of the limit states which of the limit states have been violated or which of the limit states are negative.

So, the first one is the bending limit state so if the bending capacity is less than the bending demand I increase the flexure failure count by one and I set the flag one up and then the shear limit state similarly if shear capacity is less than shear load then I increase the shear count by 1 and I set the flag 2 up and finally for the deflection limit state similarly. And this lets me keep track of both the whole system failure as well as the individual failure modes.

If any of the flags are up I add the total system failure by one. So, that's count any is incremented by the max of the flags. In the next slide we are ready to output all the results. So, the total failure probability is the count divided by the number of simulations I find the equivalent beta through the normal inverse function. I also find the estimated COV through the estimated total P f and then I output the results on the screen i do the similar thing now for each failure mode.

So, the flexural failure count gives me the flexural beta and the professional variation of the estimate and I print the results on the screen do it the same thing for shear failure and finally for deflection failure and when we go to the next slide we will study the results. So, let us take a look at these results we have these seven random variables as i described 3 limit states. So, on the left set of columns in light green background we have the flexural limit state.

Next 3 sets are in a light pink background is the sheer and the light blue background the deflection and the very last set of columns is the system limit state on the very left column I have the number of Monte Carlo simulations. So, if I have about 10 to the power 5 simulations then let us see the results the estimated flexural beta is 3.63. However the COV of the estimate is rather large let us more than 25% in shear the limit state is.

So, far back in the in the standard normal space if we may use that analogy no failure is detected. So, the failure probability is probably several ordered less than 10 to the -5 the deflection limit state has a lesser failure probability than flexure which makes sense the equivalent beta is about 3.69. The uncertainty is quite large let us about 30% for the entire system which is the union of these 3 individual failure modes the estimated beta is 3.53 and the uncertainty is about 21%.

We should have a little lower uncertainty there are COV of these estimates. So, let us now increase the number of simulations to one million you see the beta is still is in those same ballpark figures. So, for flexure let us 3.57 for deflection 3.67 for shear still no failure observed and for the system let us 3.47 the uncertainty is all less than percent. So, we could actually have stopped here but you know let us go on just because we can.

So, 10 million and then 100 million simulations and the uncertainty comes down to less than 1%. So, clearly the system beta is 3.45 the system sorry the flexure beta is 3.55 and the deflection beta is 3.7 and no shear failure has been observed. So, we really do not have any estimate there if we really want it we could do a form analysis of the shear limit state and obtained an estimate of the beta. If one is interested such an exercise could be undertaken.