Structural Reliability
Prof. Baidurya Bhattacharya
Department of Civil Engineering
Indian Institute of Technology, Kharagpur

Lecture –215
Capacity Demand Systems Reliability (Part 06)

We continue with our three lecture sequence on structural systems reliability. In the previous lecture we introduced the concepts and started looking at the series system. We started with the simplest possible configuration a two unit load sharing system and we studied the effect of dependence among those two unit strengths as well as uncertainty in the load that they share. As I mentioned looking at serious systems is not only instructive but it is also very useful because many problems in structural systems reliability actually turn out to be serious systems.

If not physically then at least logically.. So, with this lecture we are going to look at one such situation where the logical arrangement of the system is a series.

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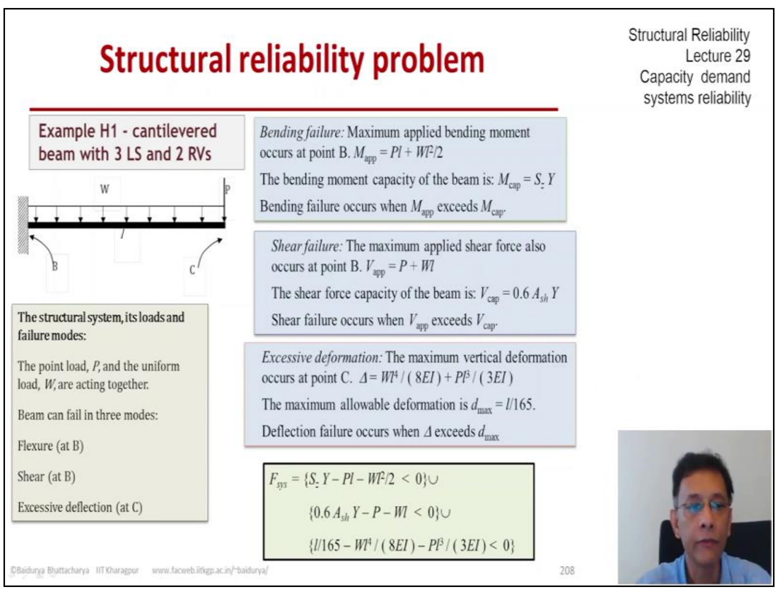

We have a very simple structural member a cantilevered beam and it has three failure modes. So, the free end is C where there is a point load P, W is the uniform distributed load and we would like to look at bending failure shear failure and excessive deflection failure. Now our knowledge of mechanics tells us that if bending failure has to occur it is most likely to occur at the fixed support beam because that is where the highest bending moment occurs under this sort of load and likewise if shear failure has to occur it will occur at B where we have the maximum shear force.

Again and finally if deflection has to be looked at the largest deflection will occur at the free end C. So, this way we reduce the dimensionality of the problem considerably. Also it is clear that now we are looking at a logical arrangement of three failure modes which give rise to a serious system as we will see. First let us look at bending failure. So, the maximum applied bending moment comes simply from statics it is P l + Wl squared by 2.

The bending moment capacity looking at plastic section modulus is S z times Y, Y being the yield strength and bending failure occurs when the applied moment exceeds the moment capacity. Coming to shear failure the applied shear at point B is P + Wl. The shear force capacity is 0.6 times the area resisting shear times the yield strength and shear failure occurs when the applied shear exceeds the capacity.

For the excessive deflection we do a linear elastic analysis and the deflection at point C is Wl to the power of 4 over 8 EI, E being the Young's modulus and I being the moment of inertia + Pl cubed over 3 EI and we allow a maximum deflection of a span over 165. Putting all of these three together we have the union of three failure events and that is why we call it a serious system because failure in any of these three modes if any of these three limit states are exceeded we have system failure.

Now we are going to solve this problem and see the results or learn some of the different aspects of it through plots and graphs. So, we have chosen only two of these quantities to be random variables the two loads all the other quantities the geometric quantities the strength type quantities we have for the time being left as non-random. But later on in the next example we are actually going to look at all of them being random variables and also we have some dependence among them.

And the way that we have chosen these numbers for this particular problem is to just make sure all the limit states are captured nicely on one single graph. So, this number 165 is actually a result of that. So, let us now define all the numbers in the next slide.

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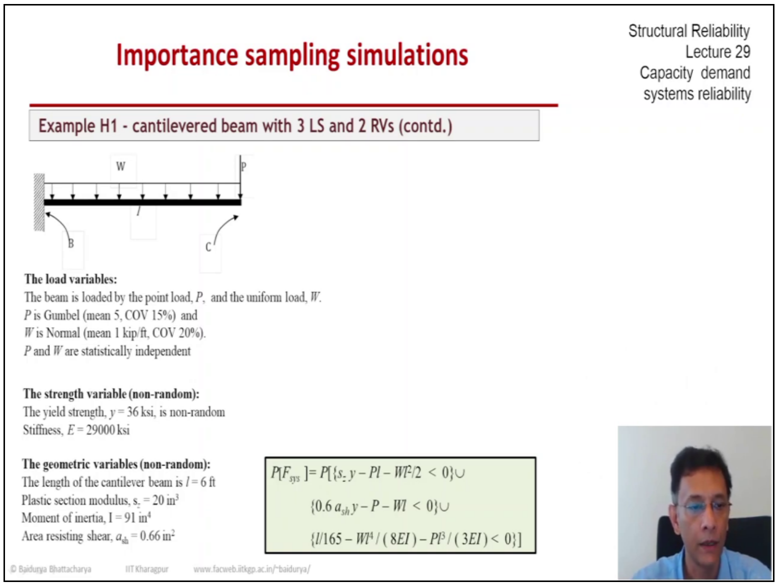

So, let us first introduce the load variables let us say we the point load P is gumball with mean 5 kip that that unit kip is missing and the distributed load is one kip per linear foot with a cog of 20% and let P and W be independent later on we are going to consider dependence among them in the next example. The strength variables for now we have them as non-random. So, the yield strength is 36 ksi and the stiffness is 29000 cases. So, we are using non-SI units for this problem.

Finally the geometric variables also non-random as I said the span length is 6 feet the plastic section modulus is 20 cubic inch the moment of inertia is 91 inch to the power 4 and the area resisting shear is 0.66 square inch. So, with this now let us display the problem graphically and study the limit states in the basic variable space.

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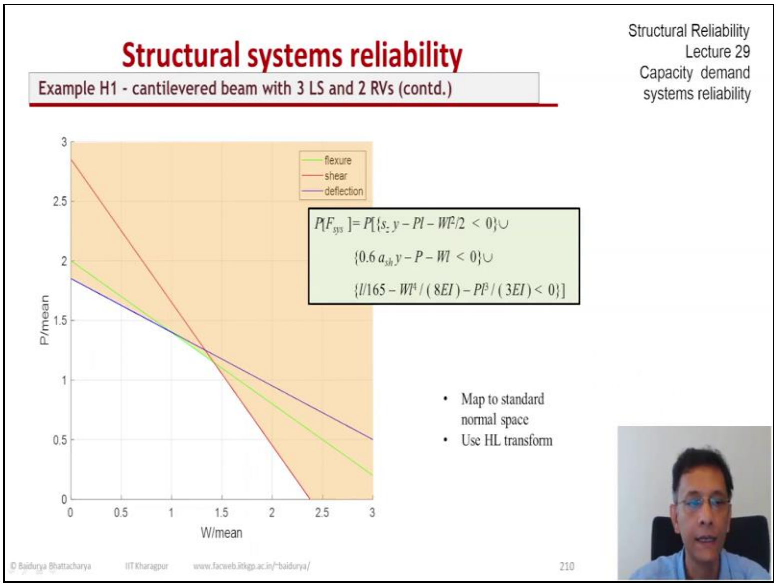

So, with all these numbers we can plot these three limit states on the space of P and W. So, W is on the x axis normalized by its mean value and P is on the y-axis normalized by its mean value and you see that we have plotted the three limit states and clearly the limit states are linear in these two variables whether it be shear or flexure or deflection P and W appear just in their linear forms. So, we have three linear limit states the green one is flexure of bending the red one is shear and the blue one is deflection.

Now because this is a series system that is how we have defined failure a failure of the system is failure in any of these three modes. So, we have the union of these three limit states and the failure region if we were to identify it on this PW space it is going to look like this. So, we have taken the union of the individual limit states. So, you see in the left portion it is governed by deflection in the middle portion it is governed by flexure and in the lower right portion it is governed by shear obviously this sort of relationship would change depending on the changes in the geometric properties changes in the allowable limits and so on.

So, as I said for the purpose of this graphical representation the numbers were chosen. So, as to have all of these things kind of be in the same region so that we could nicely see the effect of the system interaction we also see that these all these limit states kind of are dependent on each other because they their slopes are more or less in the same range and we will see what it means in the next few slides.

So, the next slides will have the map of the basic variable space onto the standard normal space why because we are going to now solve this problem using form. So now we are going to solve a system reliability problem with the help of first order reliability method which we have already looked at in the context of component reliability. And we are going to use the simplest possible transform the second moment transform or the pacifier lin transform which means the distribution of P being gumball.

For example is not going to be used we are just going to use the first two moments of that random variable. So, let us do that and we have done this many times in the context of component reliability.

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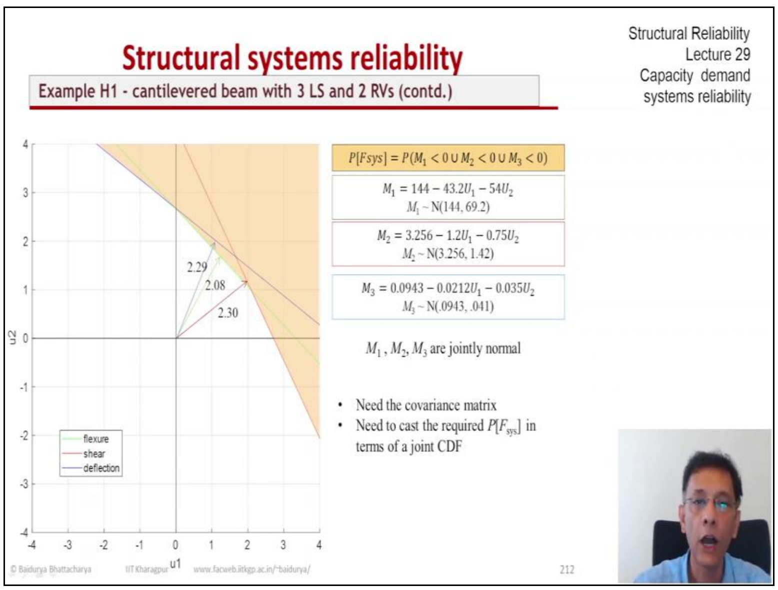

So, this is what the three limit states look like transformed onto the standard normal space of u 1, u 2. So, u 1 and u 2 are standard normal independent random variables. So, the green is flexure the red is sheer and the blue is deflection limit state and if we again shade this is what the system failure region would look like. So, now we have three optimization problems at hand each of these limit states will have their own data points and their own beta values. So, let us go through that step by step.

So, the first one the m1 the safety margin corresponding to bending or flexure is as we showed in the first slide on this problem is given in terms of P and W. And P and W are now expressed in terms of the corresponding values of u 1 and v 2. So, we end up with a linear equation in u 1 and u 2 as you see it is 144 minus 43.2 u 1 - 54 u 2. So, just to check at the origin u 1 and u 2 are zero we have m 1 positive. So, the origin is in the safe region if we want to find the minimum distance point to this limit state that is the arrow that you see.

So, that is the minimum distance point from the origin its length is beta and it is cosines are the sensitivities which actually we are going to use later on uh. So, the value for this flexural limit state beta is 2.08 and the optimal point is approximately 1.30 and 1.63 in terms of u 1 and u 2. The red line the shear limit state also we know its form once we transform P and W to u 2 and u 1 we get a linear limit state for m 2 which is 3.256 minus 1.2 u 1 - 0.75 u 2 and that is the minimum distance point with the red arrow that you see on the figure.

And the value of beta for the red line is 2.30 and the optimal point is 1.95 and 1.22. And likewise we could analyze the deflection limit state again it's linear in u 1 and u 2 and we have seen the forms and again let us make sure that m3 is positive when u 1 and u 2 are zeros the origin is in the safe region just to make sure and the linear form of m 3 in terms of u 1 and u 2 are there at the last line of that blue block.

And we can also locate its minimum distance point which is that blue arrow that you see on the left and the distance is 2.29 and the optimal point is 1.18 and 1.97. So, this way we actually have solved the three individual limit states for flexure shear and deflection. Now we need to combine them and that is the new challenge which we have not solved yet. But what do we need now? We need to we need to realize that this system failure probability which is P of m 1 less than 0 or m 2 less than 0 or m 3 less than 0.

And m 1 and m 2 and m 3 they are all normal random variables we see their statistics m 1 is normal 144 mean and 69.2 is the standard deviation likewise m 2 and m 3 they are each normal but they are dependent they are dependent because each of them is a linear function of the same two variables u 1 and u 2. So, even though u 1 and u 2 are independent m 1, m 2 and m 3 are mutually dependent.

So, we need we need to find their dependence if we can if we have to find P of f's. So, we need their covariance matrix and we also need to as we know to cast this required P f sys in terms of a joint CDF because all the joint normal distribution values are presented in terms of joint CDF's and as does MATLAB. So, we need to somehow cast this as a problem of joint CDF. So, we need intersection of events not union of events. So, let us see how we are going to do that.

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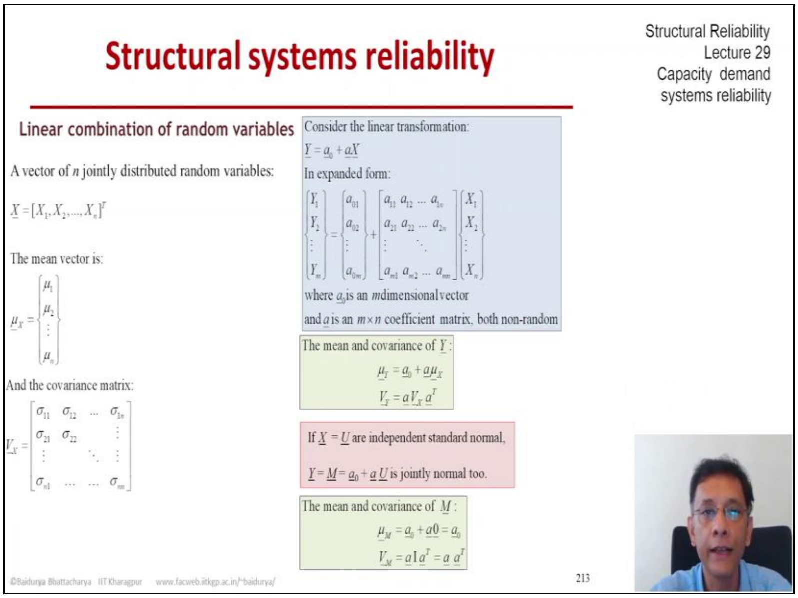

To do that we need to go back to what we learned earlier in part a of this course linear combination of random variables. So, let us recall a vector of n jointly distributed random variables X whose mean vector is mu X mu 1 up to mu n whose covariance matrix is V x and n by n square matrix. If we make a linear transformation of this y a new vector whose size is m given by a 0 + a X, a 0 being a column vector of size m and a being a rectangular matrix of size m by n. So, then we have a new random vector Y and its mean and covariance matrix.

We know is as you see on the screen the mean of Y is given in terms of the mean of X the covariance matrix of Y is given in terms of that of X with the help of a 0 and a. Now in the special case that the X's are independent standard normal. So, X is identical to u then and if we give the name M to Y. So, then this M is a joint normal random variable which is what we are trying to get at and the mean and covariance matrix of M is now then going to look like the mean of M is that constant a naught that vector and the covariance matrix of m is simply a a transpose because the V of x is now the identity matrix.

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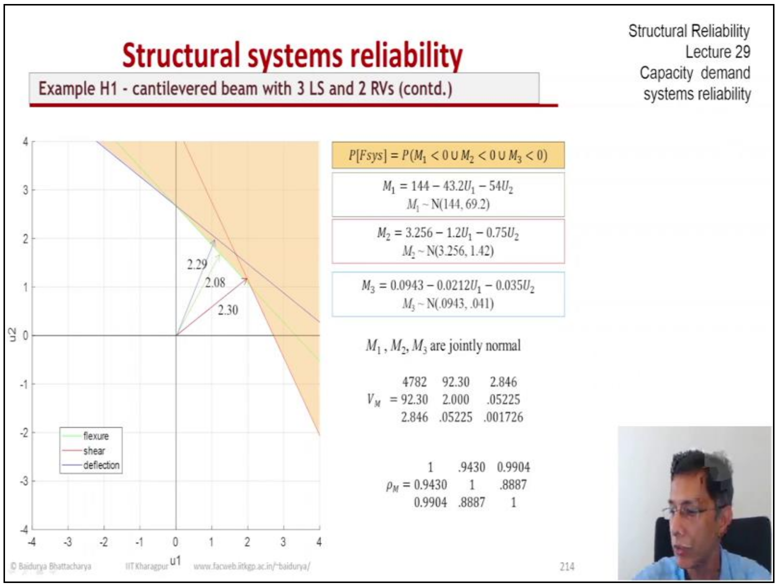

So, we need to remember this and with this knowledge we can go back to our cantilevered beam problem and derive the covariance matrix and the mean vector of M, M being M 1, M 2 and M 3 the three safety margins. So, these are the individual beta values this is the system reliability these are the M's. So, the after doing the computation the covariance matrix of m is what you see on the screen that the 3 by 3 asymmetric matrix and just for our information just for our interest.

Although we do not need this for MATLAB the correlation matrix looks like this and it is quite interesting to see that all the failure modes are highly correlated this is what I alluded to earlier and that is because you see they are quite closely linked they have their slopes around the same values and the correlation coefficient is basically the cosine of the angle between the three minimum distance lines. So, the cosine of the angle pairs between the blue, red and the green arrows actually are the correlation terms.

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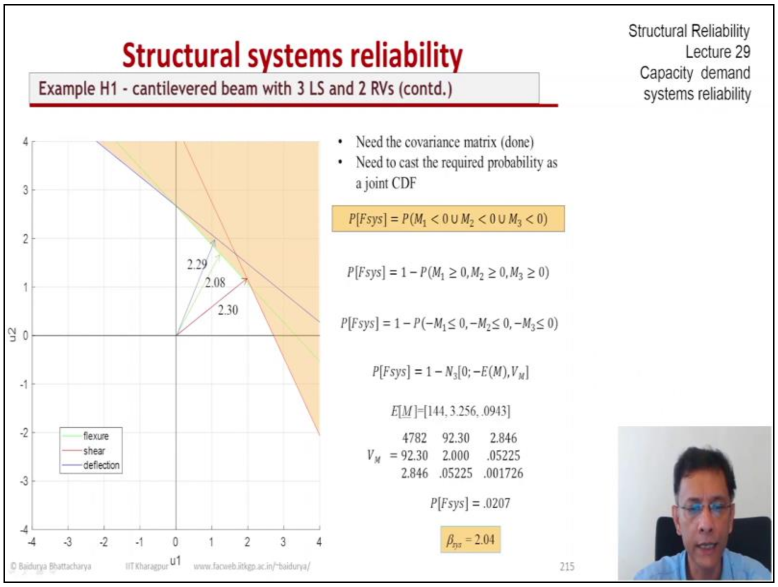

So now we are in a position to solve this problem using MATLAB the M V and CDF command provided we can express this problem as a in terms of a normal CDF. So, we needed the covariance matrix we have done that we now need to cast this problem this problem this P f sys in terms of a joint CDF. So, we use we convert the union probability using De Morgan's law to intersection probability.

So, P f sys now 1 - P M 1 greater than equal to 0 and M 2 greater than equal to 0 and M 3 greater than equal to 0. So, now we have intersecting events. So, that is good but they are greater than type. So, we have to convert them to the less than type. So, which is obvious M 1 greater than equal to 0 is minus M 1 less than equal to 0. So, we convert that so now we have reached the desired form except that we now have minus M 1 minus M 2 minus M 3 which should not be a problem because the mean of minus m is minus of the mean vector of m.

And it does not matter whether it is plus or minus the covariance matrix remains the same. So, the P f sys is one minus the three-dimensional normal integration at zero with the mean vector minus EM and the covariance matrix VM as we have already derived. So, these are the two the mean vector and the covariance matrix and MATLAB gives us the value of 0.0207 which if we do the normal CDF inverse comes to about 2.04.

So, it is interesting to note that our system reliability is 2.04 which is obviously as it should be less than all the individual beta values 2.08, 2.29 and 2.30. And we can also see that the failure mode that is most dominant governing is the bending mode which is typically what we see for beams of this type. So, this brings us to a nice inclusion of this system reliability problem of a single structure member.

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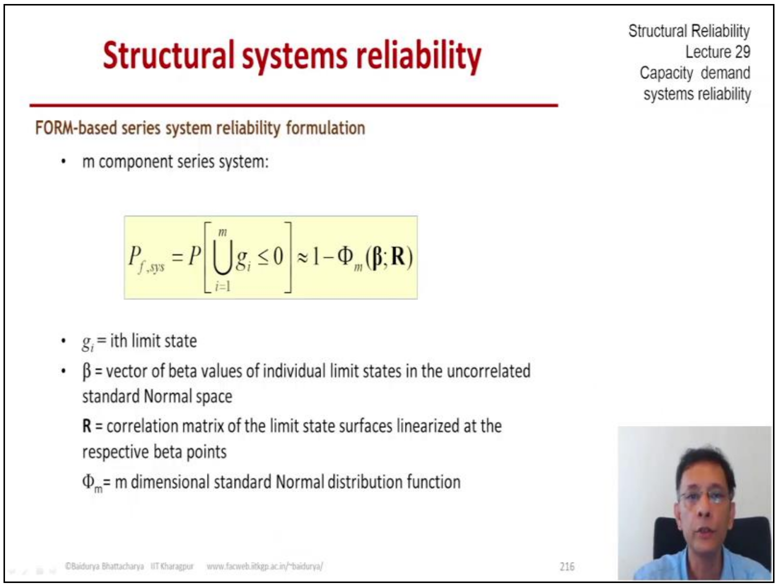

Now if we have to generalize this and use form to solve this sort of series reliability problem this is what it looks like we have the union probability of M limit states and that as I said could be given in terms of one minus a joint CDF. A joint normal CDF and now we are using joint normal CDF with zero mean and with unit variance. So, standard normal CDF that is 5 subscript M that is evaluated at beta.

Beta is at the individual element limit states the individual g i less than zero that or g i equal to zero that is the each of the betas and R is the correlation matrix not the covariance matrix because we are talking about standard normal. Now R is the correlation matrix and if we have the means either tables formulas or a software such as MATLAB we can find the system failure probability and from that the system reliability index.