Structural Reliability
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Lecture –214
Capacity Demand Systems Reliability (Part 05)

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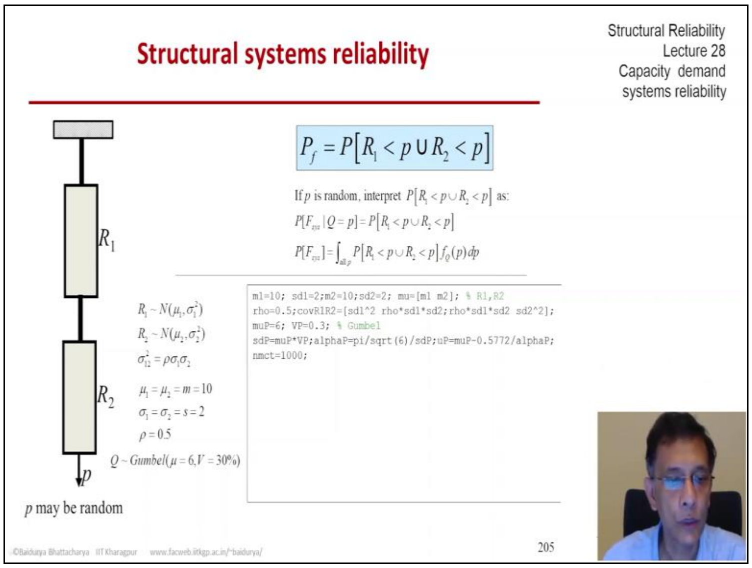

In our two unit series system where the two elements are loaded by the same load p and the failure probability is p of R 1 less than p union R 2 less than p. So, it is a union probability what happens if p is random. So, if p is random then we could interpret the P f that you see above p R 1 less than p and R I mean sorry R 1 less than p or R 2 less than p as a conditional probability. So, our P of f says given Q the random load has taken on an arbitrary value p non-random that is what we have already seen what we already discussed at length in the previous slide.

So, we have done this sort of thing quite extensively in the last few lectures. So, we invoke the theorem of total probability and get the unconditional F sys probability, probability of system failure as the integration of the kernel which we already had with respect to the PDF of q the random load. So, this could be computed without any problem using Monte Carlo simulations and we have been doing this.

So, it should not be difficult now but let us walk through an example and see what sort of effect this introduction of the third random variable randomness in the load would cause. Obviously we should expect that the failure probability should go up because now load is also uncertain. Let us see how that looks like through an example and we will continue with the example we were discussing in the previous slide.

So, R 1 and R 2 are joint normal with the correlation coefficient and we will fix the correlation coefficient at 0.5 it is a reasonable number that is all. And the means and the standard deviations are the same as before the means being 10 and the standard deviations being two as we had in the previous slide. Q we now is we say it is a random variable let us pick the gumball distribution for Q let us say its mean is six. So, it is one of the four cases that we looked at in the previous slide and it is that curve on which we made that green oval which I wanted us to remember.

So, the mean is set at 6 units and the coefficient variation is rather large 30%. So, that is our Q. Now it is most straightforward to write a small program and do this and let us see how the code would look like in such a situation. So, I present the main lines of that of that code. So, I first define all the properties the means and the standard deviations for both the random variables and in fact I want to go through this in little detail in some in some detail because we need to know how to write the syntax for MATLAB when we are discussing when we are going to call the bivariate normal CDF.

So, we need to define the mean vector which is what you see mu being m 1 and m 2 in a rho vector. The correlation coefficient rho is 0.5 and now we have to define a 2 by 2 a square covariance matrix. So, that is what we do in the second line of the code I have put the rho's next to each other it is not necessary I just wanted to save some space and fit everything in one slide. So, the covariance matrix for R 1 and R 2 the first diagonal element is sigma 1 squared.

So, that is what you see as d1 to the power 2. then the off diagonal term is the covariance term which is rho sigma 1 sigma 2 and then we come to the next line after the semicolon that is again rho sigma 1 sigma 2 and the final term the second diagonal element is sigma 2 squared. So, for any larger problem and if it is jointly normal we need to carefully define the covariance matrix because that is what MATLAB needs.

Then we come to the gumball random variable its mean is 6 its COV is 0.3. So, if I have done it correctly we have the SD the standard deviation of p the alpha of the gumball distribution and the u of the gamble distribution also in terms of the mean of alpha. So, assuming that all that coding is correct I can now start the multicolor simulation. So, you see 1000 here but actually I have done about 100 times more and the answer that you will see corresponds to about 100000 simulations uh.

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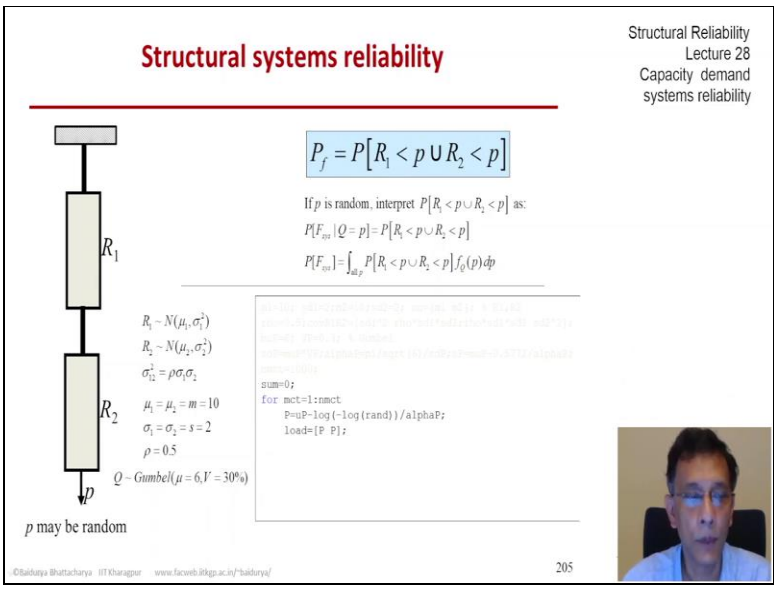

So, the next is we enter the Monte Carlo loop initialize the sum because after all we are computing an expectation as you see on the integral above p f says is integral of the union probability multiplied integrated with respect to f Q p dp. So, it is that expectation we are computing. So, we generate p every time because the expectation is with respect to p. So, the line the first line inside the Monte Carlo loop generates a value of the gumball load p.

And then because each of these random variables R 2 and R 2 is loaded equally that is why the load vector which is again something that will be required by the MATLAB by the normal CDF function call and that would require the load vector or the vector at which I evaluate the bivariate normal. So, that is P and p. So, it comes from the mechanics the problem if there was any other value of the individual member loads that's what we would put.

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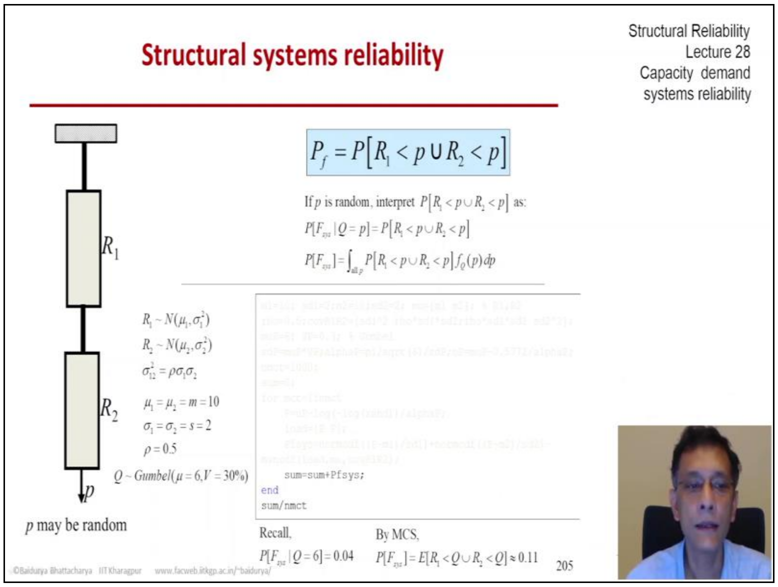

Now we are ready to call the normal CDF functions. So, we go the same way as we did before P f is P A + P B – P AB or p R 1 less than p + R 2 less than p - p R 2 less than p and R 2 less than p. So, that is what we are doing the first term is the one-dimensional normal integral the second term is the one-dimensional normal integral both evaluated at p and the third term the subtracted term is the bivariate normal CDF.

So, it is important we learn this syntax because we are going to use this for larger problems as well. So, it is mg and CDF evaluated at the load vector where mu is the mean vector and COV R 1 R 2 is the covariance matrix of the byway distribution. And once we are done with that we keep adding that estimated P f says to the sum and then when we are out of the loop then we take the average and that is all there is to it as this is what I wanted us to remember that when Q is exactly 6 the mean value no uncertainty.

So, P f sys was 0.04 approximately and with this after about 100 000 simulations the answer is the expectation that we computed is about 0.11. So, there is a substantial difference which should be expected but this tells us that we have the ability to include any sort of randomness provided we understand the mechanics of the problem and provided we are able to include dependence where it should be and then go through systematically and get the final answer.