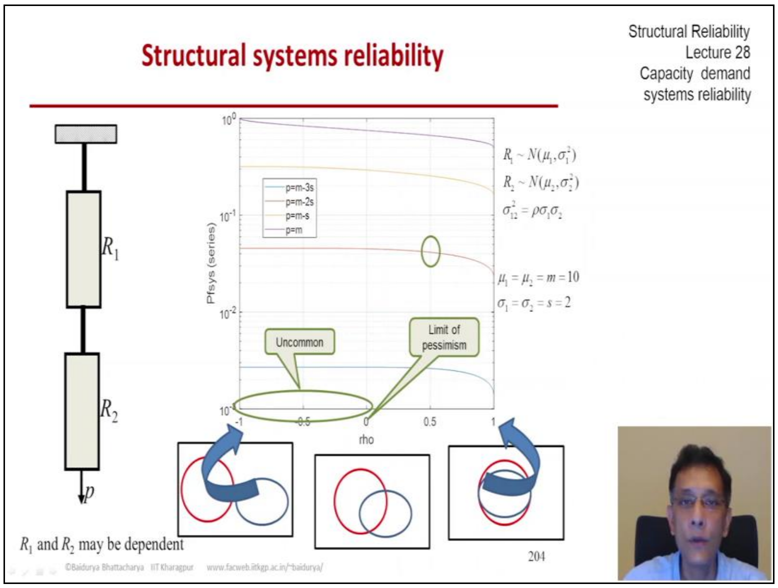
Structural Reliability
Prof. Baidurya Bhattacharya
Department of Civil Engineering
Indian Institute of Technology, Kharagpur

Lecture –213
Capacity Demand Systems Reliability (Part 04)

(Refer Slide Time: 00:54)



Let us work through an example and try to put some hard numbers to the discussion that we are having. Let us say that R 1 and R 2 are jointly normal and we allow a dependence between them and the correlation coefficient is a very useful way of introducing dependence especially between two joint normal random variables so that is what we have done here mu 1 is the mean of R 1 mu 2 is the mean of R 2 sigma 1 squared and sigma 2 squared are the respective variances and rho is the correlation coefficient.

What we will do is we will let row vary over its entire range - 1 to plus 1 and see what sort of conclusions we can draw and what sort of situations different values of rho or different ranges of rho correspond to. In particular let us say we take the mean as 10 for each of them for each R 1 and R 2 so they are identically distributed and each standard deviation is two. So R 1 and R 2 are identically distributed but they are dependent as I just mentioned.

Now if I look at the joint probability of R 1 less than p or r 2 less than p which is our definition of failure probability series failure probability. So that is what you see on your screen. I have taken four different values of the load. So the load as we mentioned is non-random at this point. So, I start with the load set equal to the mean of the individual elements mean strength of the individual elements the mean - one standard deviation.

The mean - two standard deviations and finally the mean - three standard deviations so that is what you see the top line the purple line corresponds to the case where the load equals the individual mean values. And then at the very bottom the blue line corresponds to the load equal to mean - 3, standard deviation. So that is it it starts with 10 and then 8, 6 and 4. So, 4 units is the smallest value of the load taken which is the blue line.

So, you see the entire range of row on the x axis starts with -1, full negative dependence and ends with 1 which is full positive dependence and in the middle rho 0 which for joint normal is equivalent to saying that they are independent. So, we go through the entire range now what is interesting is that on a log scale for the for the y axis you see there is huge variability or the huge effect on the value of the load which should be expected.

So when the load is quite high equal to the mean of each element strength the failure probability practically starts at 1 and then drops off to something like 0.5 with increasing rho for a lower value of load the failure probability starts at something like 0.3 and then falls to something like 0.1819 and we see the same kind of trend for the other two curves as p decreases p is m - 2s and then m - 3s we see that it starts at the highest value in the entire range and then monotonically decreases to the lowest value when rho is equal to one.

So what can we conclude from this sort of behaviour when rho is -1 it is kind of the same thing as saying that they have negative dependence and if R 1 is low then R 2 is high if R 1 is very low then R 2 is likely to be very high. It is very unlikely that both would take high values or low values together if the row is near about -1. So, that is what we what we refer to as negative dependence in the previous slide.

So when that happens we have the highest possible value of failure probability on the other hand if we say that R 1 and R 2 they vary kind of together they achieve high values together and they achieve low values together if that happens then P f says the system failure probability is the lowest in each case. So that kind of tells us that it one has to take the assumption of high dependence or full dependence a bit carefully because that leads to a low estimator failure probability and if that assumption is not right we would be underestimating failure probability which could be dangerous.

And somewhere in between, between these two extreme limits we have partial dependence or even possibly no dependence when rho is equal to zero and thereabout we see P f says almost the same as what it was on the left end of the spectrum and then it starts to fall. Now it so happens that this range the negative values of rho are really not very reasonable or difficult to defend in some sense for strengths. So if R 1 and R 2 are member strengths and these members are supposed to behave the same way we are talking about the same mode of failure tensile failure.

It is and they are presumably made of the same material coming out of the same batch may have been fabricated in the same facility. So it is quite difficult to defend the assumption or the claim that the strengths are negatively correlated. So, we typically do not go there it is a very uncommon assumption to take negative values of rho between the strings. So, we kind of limit our pessimism to rho of 0.

So although technically speaking we could go for rho - 1 but as I said no dependence if we do not know anything else about the two random variables here would be the most pessimistic the most conservative situation. And why conservative because as you see in each case rho equals 0 gives rise to the highest possible value of P of cis in each of these cases compared to all other values of rho. So, if we do not know or if we want to simplify matters it for a series system it will not hurt to have an independent assumption between different strength variables.

Provided we are talking about the same sort of failure mode. Now obviously this conclusion is not guaranteed for other configurations parallel systems for example and in fact we will see that it depending on material behaviour we are not going to be able to say something like this. So we have to be very careful about what sort of assumption we are making. Now let us just before we go to the next slide.

Let us just remember this particular number that I just circled when rho is 0.5 and when P is m - 2s so for that brown line we have a value of the series system failure probability as something like 0.04. We are going to now look at the next question that we had what if p is random and so that is what I want us to remember that the P of cis is about 0.04 and we will see what how that changes for rho equals 0.5, how that changes when we include randomness in the load.