Structural Reliability
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Lecture –212
Capacity Demand Systems Reliability (Part 03)

We start our discussion on structural systems reliability with series systems series systems are not only very instructive but it turns out that many problems in structural systems reliability turn out to be serious problems. So, they are very useful as well.

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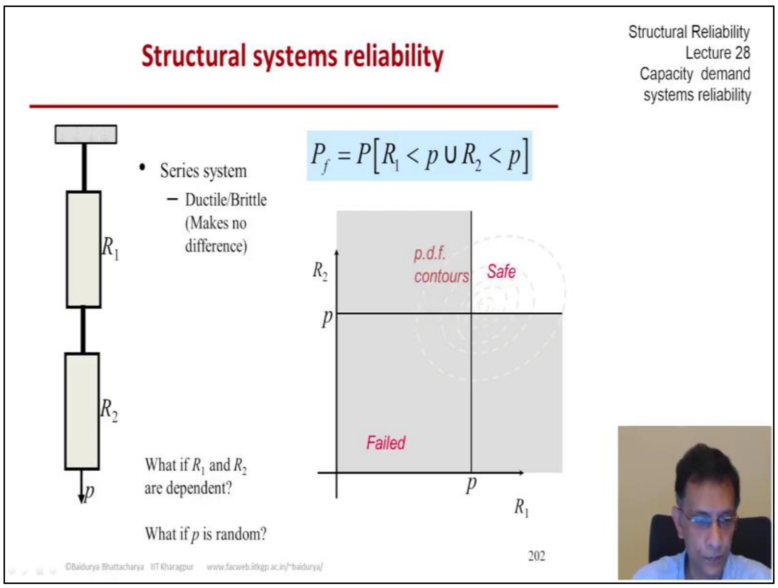

Let us look at the simplest possible configuration of a serious system a two unit system here we have a mechanical system two bars or two springs with strength R 1 and R 2 respectively loaded by an actual load p. So, clearly each of them is loaded by the same load p. R 1 and R 2 are random variables and now there would be a question what is the material behaviour are these ductile are these brittle or are they somewhere in between.

It turns out as long as we have simple series systems like this it makes no difference because the first failure, failure of any of these elements is tantamount to system failure and there is no opportunity of any residual strength or etcetera, etcetera. So, here we make no distinction whether these elements are made of ductile or brittle materials. The system failure is either of them the first unit or the second unit not being able to carry the load.

So, R 1 less than p or R 2 less than p and the system failure probability is p of R 1 less than p union R 2 less than p. We could also write we could write the reliability function as 1 minus p of R 1 greater than p and R 2 greater than p using De Morgan's law that is quite straightforward. Now what is interesting is to see what constitutes the failure region if we were to look at the limit state or if we had to integrate the joint probability of R 1 and R 2 over the space of basic variables we would need to know what is the failure region.

So, on the space of R 1 and R 2 clearly we see that the failure region is actually a big part of the entire space R 1 less than p or R 2 less than p is and it can go all the way up to infinity for either of them. So, the safe region is only when R 1 and R 2 both are greater than p. So, that is what you see on the upper right quadrant. So, if we know the joint distribution of the R's and if p is a constant then we could integrate this joint distribution.

These joint densities function and estimate the failure probability. Now obviously there could be a couple of questions here we might be tempted to just treat R 1 and R 2 as random but independent but it may not always be that simple. So, what if R 1 and R 2 are dependent in fact that dependence may come from very intrinsic causes. So, we may not be able to ignore that and ignoring dependence whether it is conservative or not conservative is something we need to figure out.

So, it is not advisable to jump to independence at the first possible opportunity. Now what if p was random we have this entire formulation that we see is based on p being a constant quantity a non-random quantity would the development change essentially or it would just be built up on that we will see that quite soon. But now let us try to answer the first question that what happens if R 1 and R 2 are dependent and what can we say in general terms about that.

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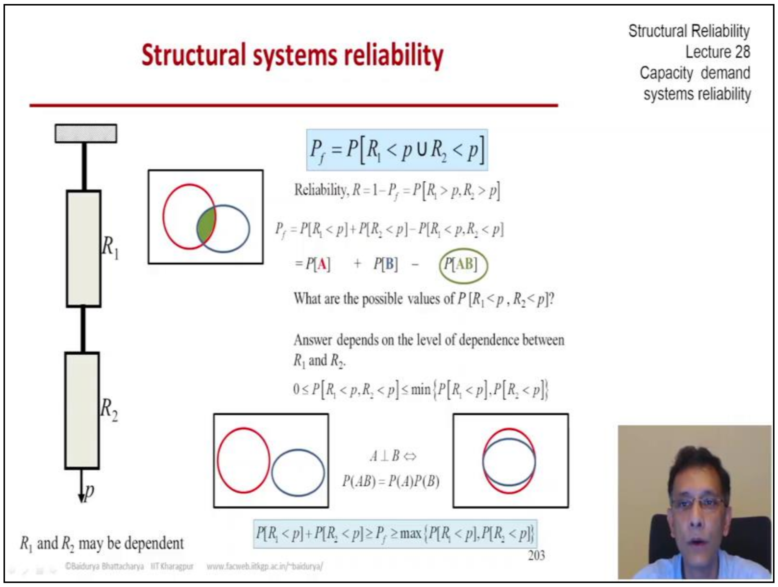

So, this is where we start with P f is p of R 1 less than p or R 2 less than p and R 1 R 2 may be dependent. So, let us express the union probability that we have at the top as the sum of the individual probabilities minus the probability of the intersection. So, the two first order probabilities which is R 1 less than p and R 2 less than p they are not affected by dependence they are basically their first order or marginal probabilities.

So, that is not of interest in this situation when we are discussing dependence what is of interest is the intersection term which we are subtracting. So, if we say it s PA + PB - PAB because we are basically expressing P of A union B. In terms of the Venn diagram the P of A is the red circle or oval P of B is the blue oval and P of AB is the green intersection that you see. So, what can we say about that or how does that intersection affect the probability of system failure.

So, that is something we are going to probe a little. So, what are the possible values of P of R 1 less than P and R 2 less than P that intersection the answer depends on the level of dependence between the two random variables R 1 and R 2. We can say without knowing anything specific about the dependence that we can say that the intersection probability is at least zero. So, it is 0 or higher. So, 0 is the lowest possible value and the highest possible value is one of the two first order probabilities particularly the lesser one.

So, the smaller of the two would be the highest possible value of the intersection probability. To show how this what this corresponds to in terms of the Venn diagram the left limit the 0 value basically corresponds to the case when A and B are disjoint. So, one way of looking at it is that they are negatively dependent if one happens if one occurs we are sure that the other did not occur. So, they are disjoint and very dependent and in a negative manner.

The other extreme is when one is completely contained in the other. So, P of A intersection B is either PA or PB depending on which one is smaller and so, that would be the highest possible value of the intersection. Between these two limits we could be anywhere and depending on exactly how much the intersection is we would have different values of P AB, P A intersection B or P R 1 less than p P R 2 less than p.

And specifically; if they were independent then the intersection would be just as big as necessary to have a probability equal to the product of the first order probabilities. So, without really knowing anything specific about the dependent structure we can at least say that for a series system like this the probability of failure would be less than the sum of the first order probabilities and would be at least as large as the larger of the two first order probabilities.

This thought actually will be useful when towards the end of this discussion on serious systems we are going to look at bounds of the system failure probability of both upper bounds and lower bounds. Because sometimes if we have many individual elements we have the union of n failure events then obviously to get higher and higher order of joint probability information may be very difficult.

So, we are going to use first order or second order or in some cases maybe third order also probability information to get some reasonable bounce but we are going to come to that later.