

**Structural Reliability**  
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**Lecture –211**  
**Capacity Demand Systems Reliability (Part 02)**

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Structural Reliability  
Lecture 28  
Capacity demand  
systems reliability


### Recap: Reliability problem formulation

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Phenomenological vs. Physics based

- Is the definition of satisfactory performance by the item:
  - Available in terms of the physics of the problem?
  - If yes, is the randomness in the physical variables known?
  - If yes, is their time-dependence relevant and known?

- **Yes** ⇒ Physics based reliability problem.
  - It is often called capacity demand reliability problem, or
  - stress-strength-time problem,
  - a special case of which is the structural reliability problem.
- **No** ⇒ Phenomenological reliability problem
  - Failure is identified by observation,
  - typically in term of time to failure (TTF)
  - which is the only available random quantity describing each component.



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We use the terms and physics based once or twice in the previous slide. So, let us just recall what those are and how they are different. So, we have a set of questions is the definition of satisfactory performance by our item available in terms of the physics of the problem? If so, do we know the randomness is involved including is there any time dependence issues there. So, if the answer is no then we really cannot do much we have to observe failure perhaps test many, many identical samples of that unit and then observe failure estimate time to failure and so on.

So, that is what we called phenomenological approach to reliability. But if the answer is yes the obviously then we would go for a physics based reliability problem especially because in the types of units that we are interested in structural components and systems we do not have the ability in most cases to do many tests on nominally identical samples. So, we are almost required to look at the underlying mechanics of the problem and derive failure from the underlying mechanics including how the elements the component failures come together to define system

failure.

So, that is what we mean by physics-based approach to failure and obviously for structure that is almost always the way forward.

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### Recap: Reliability problem formulation

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#### System vs component/element

- Is the item of interest:
  - made up of two or more units
  - Logically or physically connected
  - so that the item's performance can only be described in terms of the units' performance
  - Do I have failure data or failure model for these units?

- Yes  $\Rightarrow$  System reliability problem.  
Each irreducible unit is an element (or, component).
- No  $\Rightarrow$  Element (or, component) reliability problem

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Then we also mentioned systems. So, let us again recap how we defined what a system is as opposed to a component or an element. So, is the system sorry is the item of interest is it made up of two or more units that are logically and physically connected. So, that the item's performance whether it is functioning or not whether it is failed or not can only be described in terms of the unit's performance.

So, that is the key part if I am able to define the system's performance just with one condition then it does not matter how large the system is how complicated system is I would treat it as one component one element but if on the other hand I have no choice but to describe the system's performance in terms of its underline if it is in terms of its constituent units then we have an element or component reliability problem provided we have data or a model of failure of these constituent units.

So, in that case each such irreducible unit is an element of the system and we just need to know the logic how they come together and build up the system. And if it is not it is an element

reliability problem we use the word element and component interchangeably and we have looked enough of these. So, the focus of this week is system reliability problem. So, which means that we know the element failures we have descriptions definitions criteria for element failures.

And for our case in structures we typically call those limit states. So, let us now see how the system comes together in terms of its element limit states.

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## Recap: Limit state and failure

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$Rel(t, \Omega; \Gamma, \Theta)$  = Probability that an item occupying a logical or physical domain  $\Omega$  will perform its required function(s)  $\Gamma$  under given conditions  $\Theta$  for a specified time interval  $(0, t)$

Consider one failure mode at one critical location  $\underline{x}$ :

Failure =  $\{M(\tau, \underline{x}) \in \bar{\Gamma}_{sfg}\}$ , for any  $\tau \in (0, t]$ , for given  $\underline{x} \in \Omega$

A "component" is an item of reliability that describes one failure mode at one critical location.

For component  $i$ ,

$$\bar{I}_i(t) = \begin{cases} 0, \{M(\tau, \underline{x}) \in \bar{\Gamma}_{sfg}\}, \text{ for any } \tau \in (0, t] \\ 1, \{M(\tau, \underline{x}) \in \Gamma_{sfg}\}, \text{ for all } \tau \in (0, t] \end{cases}$$

For system, the structure function:


$$\bar{I}_{ns}(t) = \alpha(\bar{I}(t)) = \begin{cases} 0, \text{ if system failed during } (0, t] \\ 1, \text{ if system is up at } t \end{cases}$$

Component limit state equation:

$g(\underline{X}; \tau) = 0$   
such that  $g(\underline{X}; \tau) < 0 \Rightarrow \{M(\tau, \underline{x}) \in \bar{\Gamma}_{sfg}\} \Rightarrow$  failed  
 $g(\underline{X}; \tau) > 0 \Rightarrow$  safe

Probability of failure:

$$P[X \in \bar{\Gamma}_{sfg} \text{ at any } \tau] = P[\alpha[g(\underline{X}; \tau) < 0] = 0, \text{ any } \tau]$$



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Recall that we defined reliability of an item of interest as the probability that it serves its purpose satisfactorily over the entire duration of interest 0 to t under given conditions. So, this particular item of interest occupies a logical or physical domain omega. So, if we fix a particular location if it is a physical domain a particular location x or more generally if it is a logical domain some particular identifier tag of that domain then we can define failure at that location or at that identifier as some function and in our case a physics based capacity demand based function safety margin function going out of the safe set for the very first time.

So, that gives us the concept of a component or an element that it is an item of reliability that describes one single failure mode at one critical location or more generally at one identifier in the logical domain. So, for that component I we have already defined the state the state random variable the indicator function the binary random variable that if it is if the safety margin if the margin function has gone out of the safe set anytime during 0 to t.

So, it is failed its value is zero otherwise it is up and its value is one. Now the system comes together in terms of its constituent elements through the structure function  $\alpha$ . So,  $\alpha$  is a function of the entire individual component  $i$ 's state functions and likewise if the system is up at time  $t$  it is one otherwise it is 0. Now when we have a physics-based approach to failure or mechanics-based approach to failure we have a function  $g$  which is equivalent to the margin function  $m$  that we were talking about that.

If that  $g$  is less than 0 for that component we have failure if that  $g$  is non-negative we have safe performance or acceptable performance and the line or the hypersurface separating these two regions is the limit state equation. So, we express equivalently the whether the component is up or down in terms of the sign of the  $g$  function the limit state equation. Now the probability of failure then brings all of these individual component limit states.

How they come together through the structure function if we are able to know that if we understand the underlying mechanics of the system then we can describe failure of the system in terms of its element failures and we can estimate the failure probability of the system in terms of the all the randomness is involved.

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## Recap: Structural reliability – unique aspects

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- Mechanics based formulation (capacity-demand-time) of performance (safe vs. failed) is necessary
  - limit state functions must be defined
  - Significant uncertainties exist in mathematical models
  - Distributed system – many failure modes, many dofs
- Time to failure is not directly available
- Non-repairable in nature

- Active redundancy
  - load sharing
- Load path dependence
  - Different loading vs unloading behaviour
- Load redistribution after initial/successive member failure
- Presence of dominant sequences within the same cut set
- Presence of non-linearities
- Time dependence of load and strength properties
- Possible statistical dependence among system properties



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Now so, putting everything together we are talking about structural reliability and structural

system reliability this week. So, we have a mechanics based formulation which is essential because we do not have an abundance of cheap identical samples to test. So, we cannot directly get the time to failure for example. So, we need limit state functions we also need to consider all the uncertainties there are many degrees of freedom.

There are many failure modes and these systems are non-repairable in nature again this term reparability is a technical term it basically means that the item of interest cannot just be taken offline and replaced by a completely new identical replacement. So, our structures are not like that. So, from that definition they are all non-repairable we cannot afford to just pick them up and replace them.

And all our systems have this property of being active redundant there is load sharing there is load path dependence and if there is unloading involved. It does not have to follow the loading path and then if there is sequence of failures then there is load redistribution and then some of these sequences may be more likely and others may be less likely. So, we need to understand that behavior as well and near failure our structures start behaving in a non-linear manner the material starts behaving in a non-linear manner.

So, all of these things coming together including all the randomness involved bring in several degrees of challenges and interesting aspects to this particular branch of reliability which is structural reliability and in particular structural systems reliability.