**Structural Reliability**
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**Lecture –21**
**Review of Random Variables (Part - 04)**

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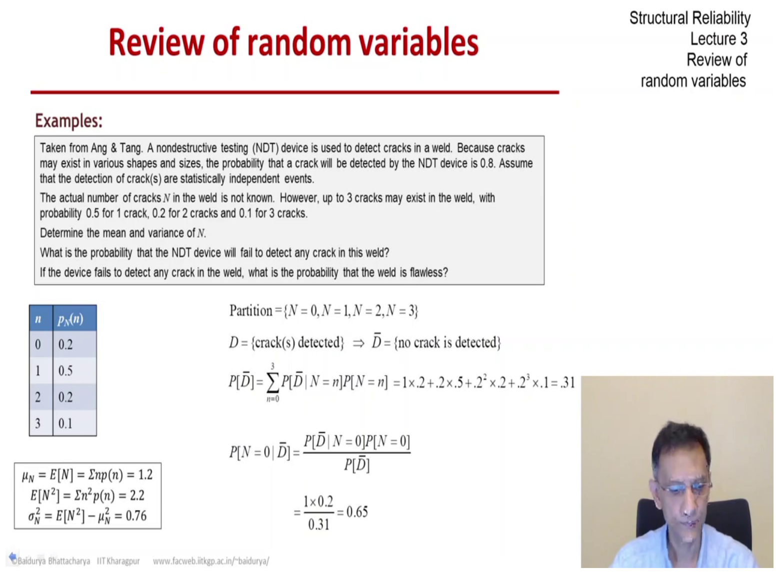


Our next example is adapted from our textbook by Ang and Tang. Let us take a minute to read the problem. So, our number of cracks is a random variable and the question has given 3 possible values but if you add the 3 probabilities they do not add up to 1. So, we need to remember that there is a possibility that the weld is flawless. So, one of the possible values of n is 0. So, if you now write down those 4 possibilities and the probabilities next to those they add up to 1.

So, the first part of the question is to find the mean and variance of n. So, let us just use the formula for the two quantities. So, the mean is the sum of n times p and that comes to 1.2 in this case if my calculation is correct and the mean of n squared would be sum of n square p and that comes to 2.2 and hence the variance which is the difference of what we just found out. So, that would come to 0.76.

So, now let us look at the second question that the device will fail to detect any crack in the weld so it seems reasonable to define a new event on top of this partition involving n. So, our partition is n equals 0 or 1 or 2 or 3 and they together they span the entire sample space and let us define D. So, these cracks may be one or more cracks are detected. So, D bar which is what actually we are interested in is that no crack is detected.

So, that is what we are going to start from. So, p of D bar using our old friend theorem of total probability would be the sum of D bar given n times p of n evaluated at the 4 possible values of n. So, let us do that one by one. So, if there is no crack. So, if n is 0, D bar is a shear event. So, that's why we have 1 times 0.2. So, the first point 2 comes from as the probability of 0 cracks in the weld. The second term is 0.2 times 0.5.

So, that 0.2 is that there is one crack and it is not detected. So, we have already been told that that a crack will be detected has probability of 0.8 which means it will not be detected as probability of 0.2. So, that is the second term the third term is if there are two cracks and both will not be detected. So, that is 0.2 squared times the probability of two cracks. So, that is again 0.2. So, that gives us a third term and the final term is there are 3 cracks and none of them will be detected.

So, that's 0.2 cubed and there is a 10% chance that there is there are 3 cracks. So, that gives me the final term and if you add all of them up you get the sum of 0.31. So, the probability that no crack will be detected is 31 the final question is that if the device fails to detect any crack then how likely is the weld to be flawless. So, it is basically switching the problem and we can use Bayes theorem and let us do that.

So, the question is asking the probability that n is 0 if D bar has taken place. So, we will use just the standard expressions. So, D bar given n equals 0 which we have already used and p of n equals 0 the unconditional probability and normalized by p of D bar and if you plug in all the values the answer comes to 0.65. So, if no crack is detected there is a good amount of chance 60% that the weld is indeed flawless.