

Structural Reliability
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Lecture –208
Capacity Demand Time Component Reliability (Part 17)

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Reliability based maintenance

Example: role of maintenance

Capacity degrades with time: $C = C_0 \exp(-t/150)$ t in years, $t_f = 100$ years.
 C_0 is Normal, mean=20, s.d. = 2; Load is time invariant Q is Normal, mean =10, s.d.=3
 C_0 and Q are mutually independent
Acceptable reliability: $R(t) > 0.95$,
Acceptable hazard: $h(t) < 0.002/\text{yr}$
Repair Options
Option 1: no repair
Option 2: repair to full strength every 20 years

Structural Reliability
Lecture 27
Capacity demand
time
component reliability



Let us take an example of time dependent reliability of a degrading system and let us for simplicity take it to be non-randomly degrading and see how reliability can be used in deciding on preventive maintenance options. So, let us take a minute to read the problem and then we will solve it. So, we have a two random variable problem the capacity degrades monotonically and both random variables are normal the acceptable reliability is 95% and the acceptable hazard is 0.2% per year.

So, we have two options that there is no repair and then we will see what happens if we decide on that option and the second option is we will repair to full strength every 20 years and the service life is expected to be 100 years. So, let us look at the two options side by side.

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Reliability based maintenance

Example: role of maintenance (contd.) Option 1 (no repair)

In the absence of maintenance:

$$R(t) = P\left[C_0 - Q_0 \max_{0 \leq \tau \leq t} \frac{1}{\exp(-\tau/150)} > 0\right]$$

$$= P\left[C_0 - Q_0 \exp(t/150) > 0\right]$$

$$= P\{M(t) > 0\}$$

Note that due to the monotonically decreasing nature of $d(\tau)$, the limit state is evaluated *only at the right end point* of the interval $(0, t)$.

The margin process M is normally distributed being a linear combination of normals. Its mean and variance at time t are:

$$\mu_M(t) = 20 - 10 \exp(t/150)$$

$$\sigma_M^2(t) = 4 + 9 \exp(t/75)$$

The reliability function therefore can be expressed as the normal CDF:

$$R(t) = \Phi\left(\frac{\mu_M(t)}{\sigma_M(t)}\right)$$

The density function is its negative derivative:

$$f(t) = -\phi\left(\frac{\mu_M(t)}{\sigma_M(t)}\right) \frac{\sigma_M(t) d\mu_M/dt - \mu_M(t) d\sigma_M/dt}{\sigma_M^2(t)}$$

Yielding the hazard function as:

$$h(t) = \frac{f(t)}{R(t)}$$



We will look at option one first we have actually solved option one type problems in previous lectures. So, in the absence of maintenance we know what to do with the safety margin function and since it is a monotonically degrading strength situation we can take the worst-case scenario at the right end of the interval 0 to t and both C and Q being normal we have a normally distributed safety margin and it is quite straightforward to find the reliability function and the hazard function and we have solved these problems. So, it should be straightforward to find how r and h behave in this option one of no repair.

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Reliability based maintenance

Example: role of maintenance (contd.) Option 2 (full repair every 20 years)

As before, for $t < t_R$:

$$\mu_M(t) = 20 - 10 \exp(t/150)$$

$$\sigma_M^2(t) = 4 + 9 \exp(t/75)$$

$$R(t) = \Phi\left(\frac{\mu_M(t)}{\sigma_M(t)}\right) \quad 0 < t < t_R$$

$$f(t) = -\phi\left(\frac{\mu_M(t)}{\sigma_M(t)}\right) \frac{\sigma_M(t) d\mu_M/dt - \mu_M(t) d\sigma_M/dt}{\sigma_M^2(t)}, \quad 0 < t < t_R$$

$$h(t) = \frac{f(t)}{R(t)}$$

Now, for $t > t_R$:

Perfect maintenance every 20 years:

$$R(20+t) = R(20)R(t) \quad 0 < t' < 20$$

$$R(40+t) = R(40)R(t) \quad 0 < t' < 20$$

$$= [R(20)]^2 R(t) \quad 0 < t' < 20$$

$$\vdots$$

$$R(mt_R+t) = [R(t_R)]^m R(t) \quad 0 < t' < t_R$$

$$f(mt_R+t) = -[R(t_R)]^m f(t'), \quad 0 < t' < t_R$$

$$h(t) = \frac{f(t)}{R(t)}$$

Simplifying:

$$h(mt_R+t) = \frac{[R(t_R)]^m f(t')}{[R(t_R)]^m R(t')} = h(t'), \quad 0 < t' < t_R$$



Now in option 2 which is full repair every 20 years let us first look at the time interval before that first repair time. So, before 20 years before 20 years things are basically as before in option one. So, we can find the reliability function we can find the density function and then we can find the hazard function. Now for t greater than t_R we have something interesting going on R of $20 + t$ prime is R at 20 times r of t prime.

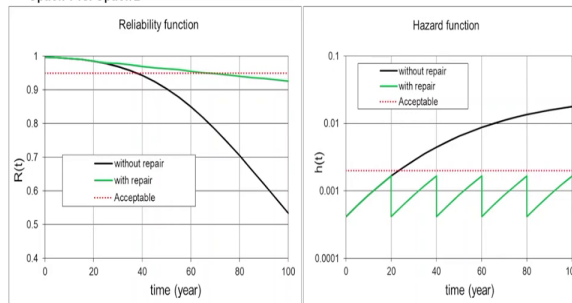
So, t prime is the interval from the last repair and because it is perfect repair we have R at $20 + t$ prime as a product of the two reliability values and it is the same way for R at $40 + t$ prime then which is R at 40 times R at again the time since the last repair which is t prime and we can do that and we obviously can use this recursively and find that R at $m t_R + t$ prime. So, after m repairs it is basically R at t_R to the power of m times the reliability as it would be in the first t prime years of its life t prime being less than 20.

So, that is how the reliability function behaves and the density function being the negative derivative of R we can get it as you see on the screen. Now the hazard function is the ratio of the two. So, we can simplify that and it. So, happens the R to the power m that cancels out. So, h of $m t_R + t$ prime is equal to h of t prime. So, h is going to be a periodic function and that is actually very interesting. Let us look for this particular example how these two options look like compared side by side.

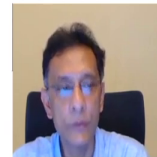
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Reliability based maintenance

Example: role of maintenance (contd.)
Option 1 vs. Option 2



Without repair: $R(t)$ becomes unacceptable around 35 years, and $h(t)$ become unacceptable around 25 years
With repair: $R(t)$ stays acceptable up to about 70 years, and $h(t)$ stays acceptable until the end



So, you see that there are 2 options in black we have what would happen without repair and in green what would happen with the repair option which is every 20 years we do perfect repair. So, and in both cases in the reliability diagram and in the hazard diagram I have drawn the acceptable limit which is 0.95 for R and .002 for h. So, if we look at the behaviour of the black line versus the green line first in the reliability and then in the hazard.

We see that the reliability is more or less indistinguishable for about 26, 27 years but then because of the periodic repair the reliability function kind of corrects itself and again starts to drop less fast the rate becomes even again the same way as it was in the beginning and it stays above the target way longer in the green case than in the black case. The hazard function as I said is periodic in nature.

So, if you do repair and perfect repair then the hazard function just repeat itself every 20 years. So, we can draw this general conclusion is that without repair the reliability becomes unacceptable around 35 years of age which is when the black line crosses the red dotted line and $h(t)$ becomes unacceptable around 25 years of age which is when the black hazard line crosses the red line horizontal? But with repair $R(t)$ stays acceptable for about 70 years which is very good and $h(t)$ stays acceptable till the very end.

So, clearly this lets us look at the effects of maintenance preventive maintenance on the time dependent reliability and time dependent hazard functions and how we can use such knowledge to take decisions.