

Structural Reliability
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Lecture –203
Capacity Demand Time Component Reliability (Part 12)

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Capacity-Demand-Time formulation


Case 3d: Random aging and Q is a Poisson pulse process

Example G4: Randomly aging bridge with Random loads (Poisson occurrence)

A simply supported RC highway bridge girder is subjected to random vehicle loads occurring according to a Poisson process. The beam is subject to random corrosion loss. Its behaviour is assumed to be linear elastic. Find its reliability function.

<p><i>Random strength parameters:</i></p> <p>(1) Concrete strength, F_c: Lognormal with $\mu=34.125$ N/mm², $\sigma=6$ N/mm²</p> <p>(2) Rebar strength, F_s: Lognormal with $\mu=456.5$ N/mm², COV=10%</p> <p>(3) Bond strength, τ_b: Lognormal with $\mu=2$ N/mm², COV=20%</p>	<p><i>Random geometric parameters:</i></p> <p>(4) Clear cover, C_c: Lognormal with $\mu=30$ mm, COV=20%</p> <p>(5) Initial diameter of rebar, Φ_s: Lognormal with $\mu=32$ mm, COV=5%</p>
<p><i>Random loads:</i></p> <p>(6) Dead Load, DL: Normal with $\mu=34.85$ KN/m, COV=10%</p> <p>Moment due to Dead Load: $M_{DL} = DL(\text{span})^2/8$ It is invariant with time.</p> <p>(7) Live Load moment, LL: M_{LL} Gumbel with $\mu=350$ KNm, COV=30%. Live loads occur according to a Poisson process with rate $\lambda = 12/\text{year}$. The live load moment magnitudes are mutually independent and each is assumed to follow the Gumbel distribution.</p>	<p><i>Random corrosion parameters:</i></p> <p>(8) Initiation time for corrosion, T_i: Lognormal with $\mu=10$ years, COV=30%</p> <p>(9) Corrosion rate parameter along thickness, K_t: lognormal with mean 0.16mm/y, COV=20%</p> <p>(10) Corrosion rate parameter along length, K_l: lognormal with mean 0.00575/y, COV=20%</p>
<p><i>Non random parameters:</i></p> <p>Span = 8 m $b = 400$ mm (section width), $d = 850$ mm (section depth) Number of rebars, $n = 5$</p>	

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Case, 3d random aging along with a Poisson pulse process model for the loads, we will continue with the example group G in which we have been looking at the reliability of a bridge girder. This one is a reinforced concrete curtain. So let us introduce the random variables and the non-random parameters as well. So, in the strength group we have the concrete strength the steel strength and the bond strength.

In the geometry parameters we have the clear cover and the initial diameter of the rebars as random variables. In the loads we have the dead load and the live load the live loads are iid nature and they occur according to a Poisson process with a certain rate. And then we have the corrosion parameter. So, that brings in the randomness in the aging process we have three aspects covered there one is the initiation time because corrosion does not start from day one it takes some time to penetrate to break down the passive layer and so on.

So, we are taking the initiation time as a random variable the rate parameter is another random variable along the thickness and a rate parameter along the length. So, we have two rate parameters there. And finally the deterministic parameters are the span length and the sectional dimensions and of course the number of rebars.

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Capacity-Demand-Time formulation

Example G4 (contd.):
(A highway bridge girder subject to random corrosion loss)

- *Effect of corrosion with time:*
 - Corrosion affects the rebar both along the length and across the diameter.
 - The strength of reinforced concrete in tension depends on:
 - the tensile force carried by the rebar which is a function of its cross sectional area,
 - the bonding force between steel and concrete which is a function of the uncorroded length of rebar.
 - Hence, an accurate time dependent model for these two variables is crucial.
 - Assume:
 - Corrosion starts from mid-span as surface cracks there are wider
 - The corroded part of bar does not contribute to bond strength.
 - Reduction of diameter with time: $\Phi(t) = \Phi_0 - K_d(t - T_i)^{0.5}, t > T_i$
 - Reduction of uncorroded length with time: $L(t) = L_0 \exp(-K_l(t - T_i)), t > T_i$

If required, a random noise can be added to the corrosion process

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How does corrosion affect the mechanics of the girder. As I said in the previous slide we account for both corrosion loss along the depth of the rebars as well as along the length of the rebars and together they affect the bending moment capacity of the beam. We assume that once initiation occurs T_i being the random initiation time the growth of corrosion along the diameter of the rebar proceeds along a diffusion control process.

Because as you see the exponent on the elapsed time $T - T_i$ is 0.5 which is a very gentle growth process and likewise along the length of the rebar the corrosion growth is also quite gentle it is negative exponential with the exponent linearly dependent on the elapsed time. If it was important if we had information we could also add a random noise in the corrosion growth process. So, this way the level of details that we include depends on the both the need and the amount of information about the process.

And that is how the underlying physics governs the mechanics and the reliability of our problem.
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Example G4 (contd.):

(A highway bridge girder subject to random corrosion loss)

Strength:

The flexural capacity at mid-span decreases as diameter of bars decreases.

The available length for development of bond decreases with time. It limits the tensile force in steel bars at midspan.

$$\text{Area of steel} = A_s(t) = 0.25 \pi \phi(t)^2$$

$$\text{Neutral axis depth} = x_u = F_y A_s / (0.36 F_c b)$$

Limit state:

$$g = M_{cap} - M_{LL} - M_{DL}$$

Flexural strength based on available area of steel (effective depth, $D = d - C_c$)

$$M_S = F_y A_s(t) (D - 0.42 x_u(t))$$

Flexural strength based on available length for development of bond:

$$M_B = \tau_b \pi \phi(t) l(t) 0.5 n (D - 0.42 x_u(t))$$

$$M_{cap} = \min(M_S, M_B)$$



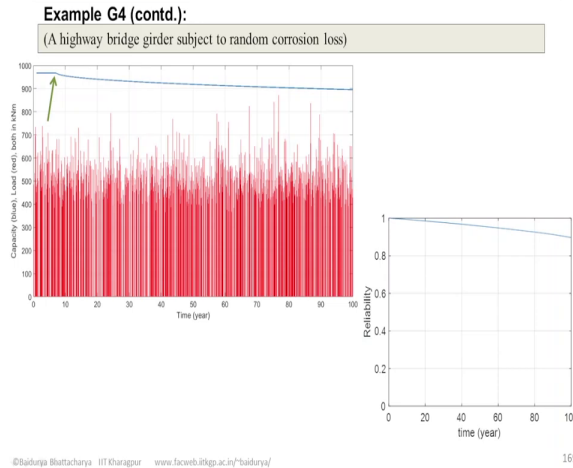
So, corrosion affects the bending moment capacity of the beam both through reducing the area of the rebars as well as reducing the bond length with the concrete. So, to be specific this is how the area of steel is affected by corrosion loss ϕ being the current diameter and that in turn affects the geometry of the section and that in turn affects the bending moment capacity based on the area of steel. And likewise the bending moment capacity based on the bond length is also affected by the diameter and the depth of the neutral axis.

So, we take the lower of these two capacities and define that as the bending moment capacity of the concrete girder and then our limit state becomes $m_{cap} - m_{LL} - m_{DL}$. So, now we are in a position to do the reliability analysis taking into account all the randomness's involved. So, as we did for one of the previous examples let us take a look at the sample function of load and strength.

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So, the red vertical lines are the gumball nodes that occur as IID pulses through a Poisson process model and the blue line that you see on the top is the just one sample function of the strength the bending moment capacity. Now what is interesting that comes out of our model is you see that until about 7 years or so, the strength is flat. So, there is no touch iteration and that is when the random initiation time hits and from that point onward the capacity starts degrading.

And as soon as one of the red vertical lines would exceed that blue line we would have failure. So, if we keep doing this through a Monte Carlo loop we would be able to get an idea of the time dependent reliability function and I have done that that simulation and what you see is the reliability function for this girder for a period of 100 years. So, this is how we will set up a problem based on the underlying physics the underlying mechanics of the problem and include all the randomnesses.

And whether the processes are stationary or not whether processes are random or not if we have the knowledge about all the random variables and random processes we have the ability to undertake a first principles based simulation and obtain estimates of the of the reliability function the hazard function and so on.