Structural Reliability
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Lecture –201
Capacity Demand Time Component Reliability (Part 10)

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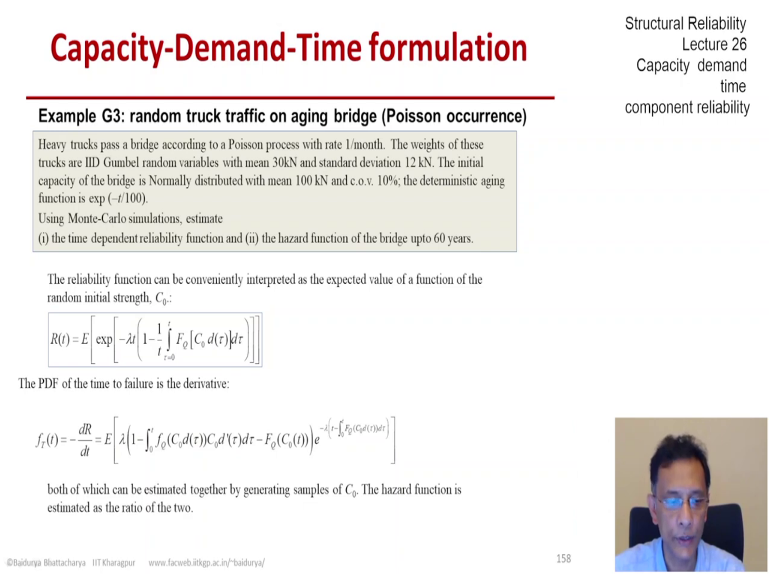

Now let us solve one problem based on case 3c. Let us take a minute to read the problem statement and then we will start solving it C. So, as you see this is the same problem G 2 that we solved before but there is one important difference that the distribution of capacity which was normal with mean 100 kilo newtons and COV of 10% is now the initial capacity and there is the aging function exponential minus t over 100.

So, it starts this d of tau of t this aging function starts at one and then falls off. In fact we will see that this is quite a severe aging and at the end of 60 years which is the life specified the mean strength falls almost to 55 of the original value. So, this is how we are going to solve this as we saw in the previous slide that the R function the reliability function is an expectation with respect to the initial capacity C naught.

The expectation is of an exponential function where we have the aging function in the kernel of integration. So, that is extra earlier when we had case 3b it was exponential minus lambda t times 1 – CDF. Now we have the area under the CDF and normalized by the length of the window in time. So, since we are asked to find the hazard function we also would need the PDF either analytically or numerically.

So here is a rather long expression for the PDF which is the negative of the derivative of r. Here if you go through the steps carefully what I have done is I have differentiated under the integral sign. So, that is how the long expression comes but we can estimate r and f this way by generating samples of C 0 and if we do that enough number of times we can estimate both r of t and h of t h being the hazard function.

So, before we present the results let us try to get a pictorial sense of what is happening with the loads and the strength.

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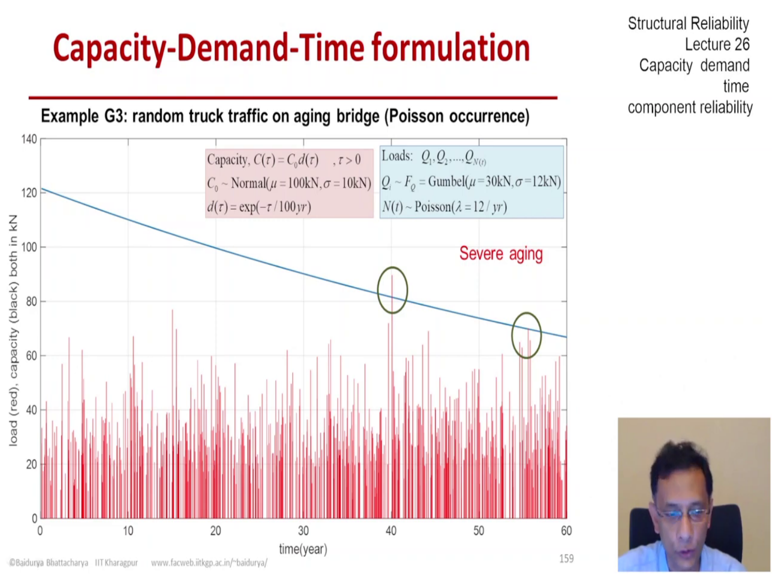

So, this is just one realization of the process the blue line at the top is the strength which is deteriorating as a function of time. So, this particular realization of the strength started at somewhere around 122 or something and then dropped off to something like 68 or something. So, and the red lines that you see the red vertical lines they are the randomly occurring loads. So, these loads occur according to a Poisson process with mean 12 with rate 12 per year that is why you see.

So, many of them and the heights of those red lines are gumbel distributed with a mean of 30k. So, it is quite interesting to see that in this particular realization of the load process and the strength process we have one clearly one failure occurring somewhere around 40 years of age. So, if we were finding the time to failure the random time to failure from this realization we would report that the TTF is 40 years.

And this is also an interesting case of the definition of the first passage you see if we allowed the process to continue then we would have another failure around 55 years of age let us say that that red line just breaches the blue line but obviously that would not matter in defining the first passage time because a failure has already happened and that's that is the end. So, we would not even continue the simulations.

So, if we do this repeatedly then we actually are going to estimate the r function and the h function we and because in this very special case we have just one random variable C 0 over which we are taking the expectation we could keep C 0 in the outer loop and let t vary from 0 to 60 and thereby get a smooth estimate of r and t of r and h.

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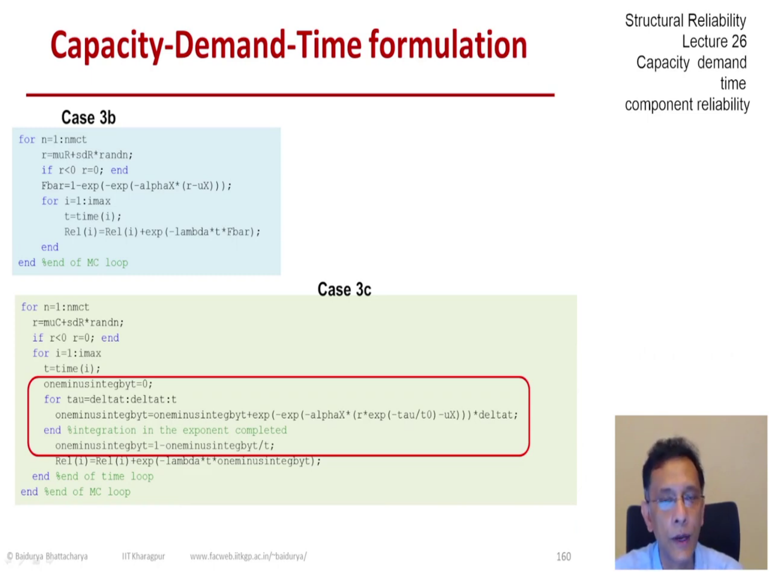

So, let us see how the code looks like just that important part of the code the code is basically the same as in case 3b where there was no aging. So, I have just looked I have just copied that part of the code which is going to change. So, if you look at if you look at the line where we compute reliability as the exponential of minus lambda t f bar f bar being 1 minus CDF of the load then that is what is going to be enhanced in case 3c.

So, in case 3c that single line has been has been replaced by that f bar computation has been replaced by an integration of the F Q evaluated at c d tau and the integration is over 0 to t. So, that that red box has the extra lines which is required to take aging into effect. So, if you do this enough number of times we are going to get the reliability function and the hazard function.

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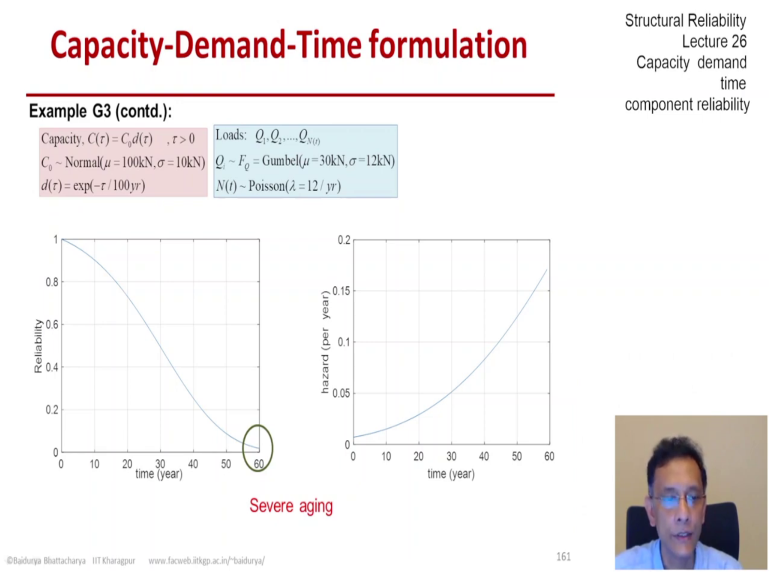

And let us see what it looks like and this again is quite severe aging and it should not be a surprise to see that for this particular structure the reliability almost falls to zero. Obviously it is not a not a real situation no one would probably build such a structure but if it was so, then we have the reliability becoming very soon very unacceptable. The hazard function can be computed and it starts from something like 0.01 roughly.007 I believe.

And then it keeps climbing if you remember in case 3b we had a hazard function that was going down we would no longer have that luxury. Now let us let us compare these results with some of the if aging was different. So, here aging is given by exponential minus tau over 100 when 1000 years. So, let us see if we have a less severe aging in the form of exponential minus tau over 1000.

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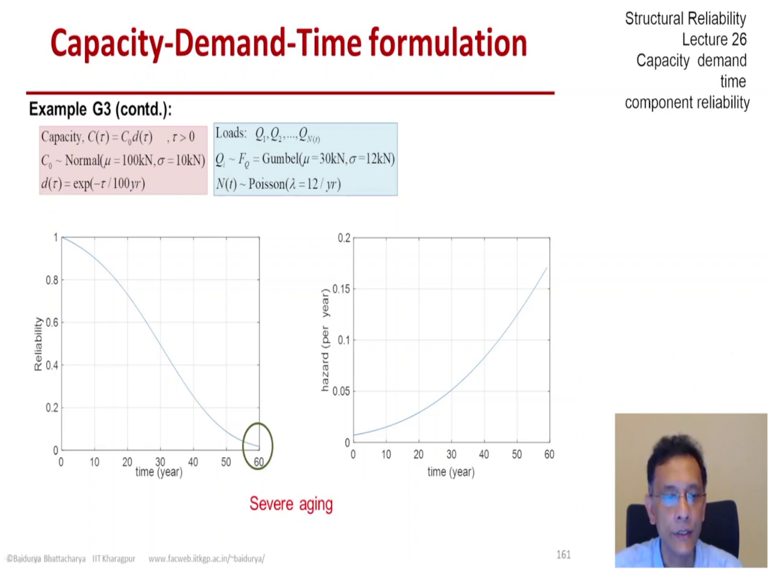

So, it is a much more benign aging situation and if we plot the reliability function we are pretty close to what we had in the no aging case the at the end of 60 years the reliability is close to 70% in case 3b we had something like 72%. And the hazard function does something interesting the hazard function initially drops. So, there is a decreasing failure rate and then somewhere around 10 or around 15 years of age it reverses scores and the effect of proof loading that was being caused by heavy drugs now is completely overtaken by the aging process. So, after about 15 years or so, the hazard function keeps climbing.

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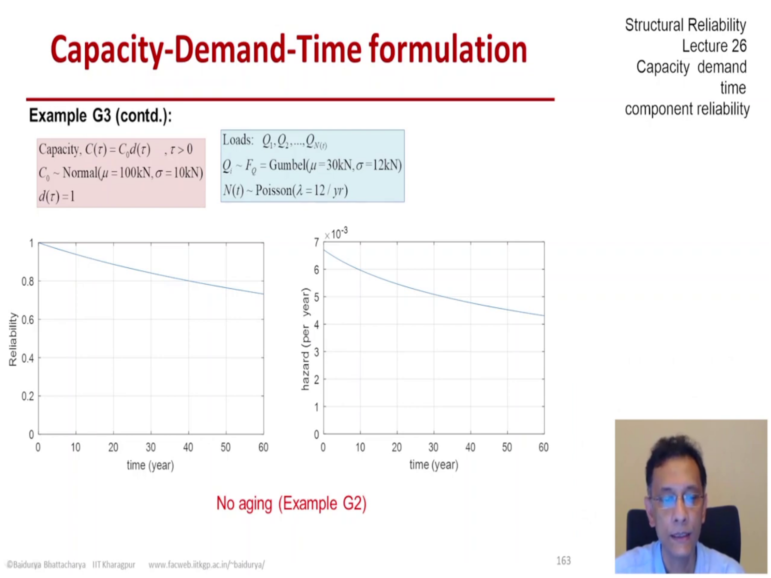

And just for completeness I have been talking about case 3b example G 2 and if we replace d of tau with 1 identically then we get back our earlier case and we have in fact the decreasing hazard function that we have already discussed.