

Structural Reliability
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Lecture –20
Review of Random Variables (Part - 03)

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Review of random variables

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 Lecture 3
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Examples:

Consider a projectile fired at an angle θ with initial velocity V_0 . The maximum range of the projectile (horizontal distance traveled) is:

$$D = V_0^2 / g, \quad (\theta = 45^\circ)$$

V_0 is modeled a random variable with mean 10 m/s and coefficient of variation 20%. What is the mean of D ?

Given: $E(V_0) = 10 \text{ m/s}$,

$\text{COV}(V_0) = 20\%$

$\therefore \sigma(V_0) = 0.20 \times 10 \text{ m/s} = 2 \text{ m/s}$

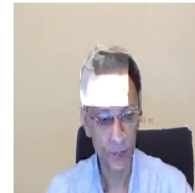
Required, $E[D] = (1/g)E(V_0^2)$

Now, $E(V_0^2) = \text{var}(V_0) + [E(V_0)]^2$

Plugging in the numbers: $E(V_0^2) = \text{var}(V_0) + [E(V_0)]^2$
 $= 2^2 + 10^2 \text{ (m/s)}^2$
 $= 104 \text{ (m/s)}^2$

Then, $E[D] = (1/g)E(V_0^2)$
 $= \frac{1}{10 \text{ ms}^{-2}} 104 \text{ m}^2 \text{ s}^{-2}$
 $= 10.4 \text{ m}$

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After this basic introduction to random variables let us take a look at a few examples. The first one involves what is known as change of variables. Let us take a minute to read it carefully. So V_0 , the initial velocity is a random variable, and there is a functional direct explicit relationship between D and V_0 . So D is also random variable. We are not interested in finding the distribution of D if we know the distribution of V_0 .

At this stage, all we want to know what the mean of D is if we know the mean and variance of the motion velocity. So let us start interpreting some of this given information. So the mean of V_0 is 10 meters per second. The COV the coefficient of variation this is a very useful engineering metric that we have for describing random variables. It is the standard deviation divided by the mean. So the standard deviation of the V_0 would be 0.2 times 10 meters per second, so, 2 meters per second.

What we need now is the mean of D . We already have seen that expectation is the linear operator, and since $1/g$ is a constant, the mean of D or the expectation of D is that constant times mean of V_0 squared. Now how do we get the mean of V_0 squared? We have been given the mean and the standard deviation of V_0 . So we will use the known relationship between the variance, which is the square of the standard deviation and the expected value of the squared random variable and the mean.

So variance is expectation of the squared random variable minus the square of the mean. So expectation of V_0 squared is the sum of two variance and the square of the mean. If we put in the numerical values, it comes to 104 meters per second whole squared. So plugging in the value of g the mean maximum range D is 10.4 meter.