# Structural Reliability


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## Lecture –193
## Capacity Demand Time Component Reliability (Part 02)

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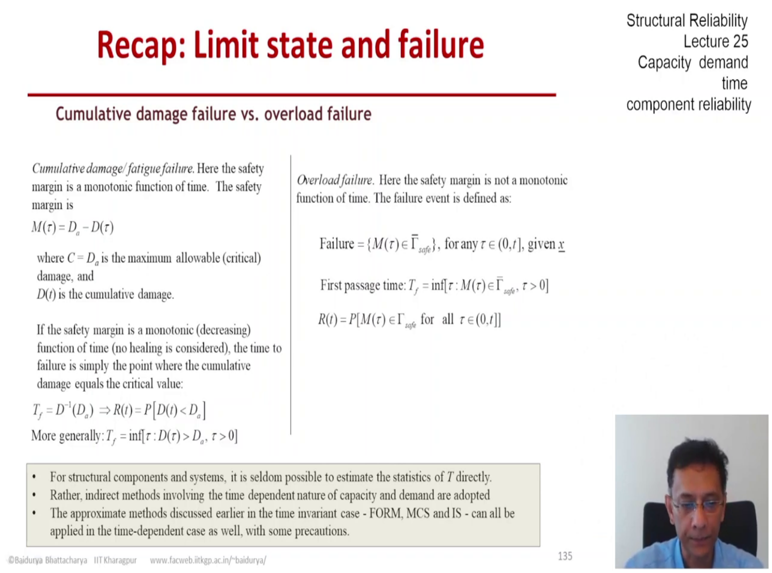


In physics-based capacity demand time formulation of component reliability we need to provide estimates of the reliability function the hazard function the random time to failure from the underlying physics or mechanics of the problem. So, in structural reliability broadly two classes of failures are recognized the cumulative damage type and the overload type in both cases the definition of the time to failure is the same.

Let us take a look in cumulative damage some kind of damage keeps accumulating within the component because of aging effects because of loading history until the accumulated damage exceeds the acceptable or the safe or the critical value. Now typically this damage growth is a monotonic process which means the safety margin keeps decreasing monotonically to 0 and as soon as that happens we can say that failure has occurred.

Now in such cases it is quite simple to define the reliability and the time to failure is that the inverse of the damage growth function taken at the critical value would be my time to failure and the reliability would be the probability that D of t is less than the critical value. So, this t is the end of the reference period that we are interested in. If there is some healing effect like crack closure effect in fatigue crack growth obviously we have to generalize this.

And the time to failure in that case would be the first time that the damage growth function exceeds the critical value. In overload type failure something very similar happens in terms of the safety margin but there is no damage accumulation taking place there is a sudden or there is a huge overload of large demand placed on the component and the capacity is exceeded and when that happens we have failure.

So, the safety margin M exceeding the safe set for the very first time is the definition of failure and gives the definition of the first passage time the random time to failure as well. And the reliability is the probability that the safety margin stays in the safe set for the entire duration that is exactly how we define in the cumulative damage case as well. So, if the damage growth is monotonic then we have simplified the reliability problem considerably.

Because we do not need to worry about all the instance of time from 0 to t but we can look at the end instant but if it is not monotonic or if we have overload type failure then we have to look at the entire duration. And so to summarize we for structural systems it's seldom possible to estimate reliability or time to failure from data as we have seen in the past unless it's a mass-produced item where a

lot of tests can be undertaken rather we have to look at the underlying mechanics of the problem and the good news is that whether we are bringing in the time aspect or not the methods that we have already looked at FORM, Monte Carlo simulations and various reduction techniques can all be applied but we need to be careful that we are doing it correctly.

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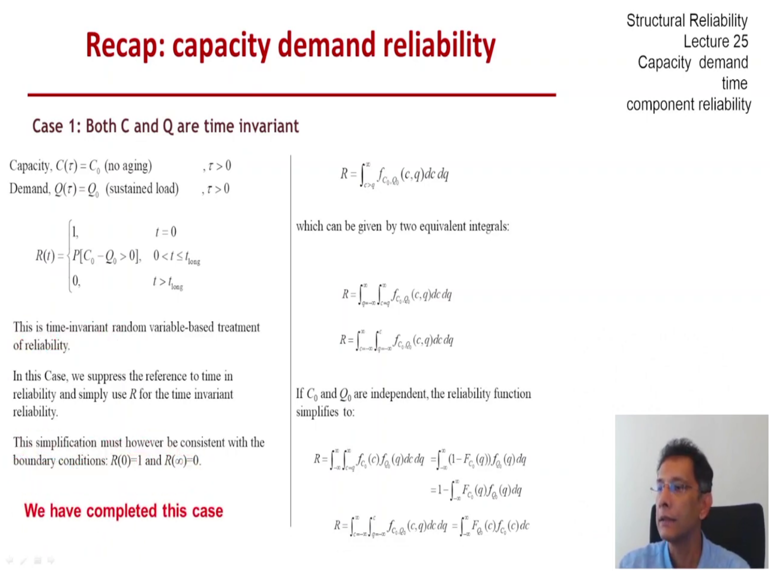


In case one which we have completed we did not consider the time to failure because both the capacity and the demand were invariant with time. So, C was a random variable C 0 at all times of interest and likewise the demand Q was another random variable Q 0 at all times of interest. So, it was a case of no aging and a sustained load but in order to be true to our definition of the reliability function that it starts at 1 at t equals 0 and falls to 0 at very large values of time uh. So, we said that R of t is equal to 1 at t equals 0 but as soon as the structure is loaded the reliability falls down to whichever value it has based on the physics of the problem.

And it stays that way for the longest time well beyond our duration of interest and then after t long it slowly or rapidly falls down to 0 but which was not of our real concern and we spent the previous 6 lectures actually discussing this setup and we learned how to formulate the problem, how to compute the reliability we have already mentioned all of that. So, we define reliability if it is a two variable problem as the integration of the joint PDF over the failure region.

And we looked at various versions of that integration whether C 0 and Q 0 were independent or not and that was our scope in case one.

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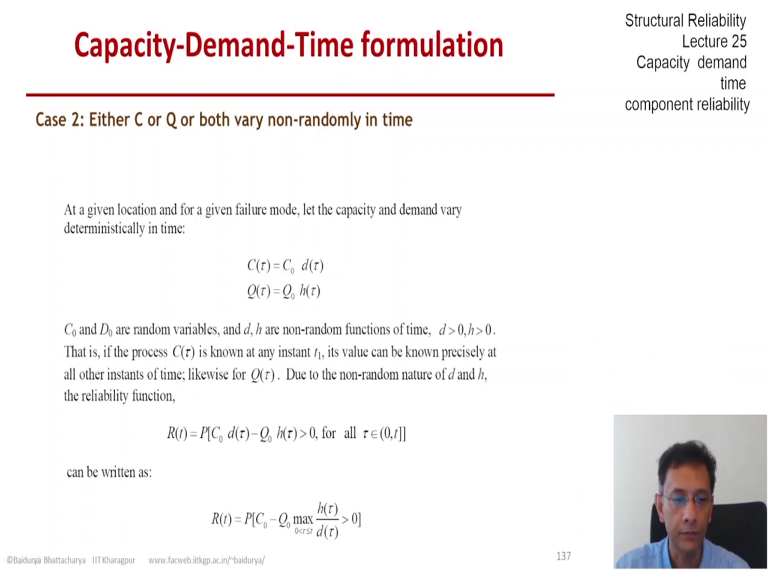


Now in this lecture we are going to look at the next case which we are calling case 2 and the way we have set it up is C or Q or both the capacity or demand or both of them vary in time but they do so, non-randomly that is important because we are we like to look at stochastic processes later. So, at this point we are just looking at a very simple version of the problem. So, this is the problem set up is we have C of tau now.

The initial random variable multiplied by a deterministic a non-random function of time and likewise the load is the initial random variable multiplied by another function of time but also non-random. And so, in the language of stochastic processes if we know the value of C at any instant of time t 1 we know it at all other instance of time precisely and without any uncertainty. So, that is the meaning that this function of time D is not a random process and C 0 is a random variable and the same goes for Q 0 and h of tau.

Now because the functions D and H are non-random we can actually simplify the reliability function considerably and it looks like the probability that C 0 times D of tau minus Q 0 times h of tau is greater than 0 for all tau. But now we can divide both sides by D of tau and express this as the probability that C 0 minus Q 0 times a factor which is the maximum of h over D over all times of interest.

So, this way we are able to simplify this time dependent problem into again a two random variable problem but there is a time angle where that ratio of h over D comes and we have to take the maximum value of that over the time instant 0 to t.

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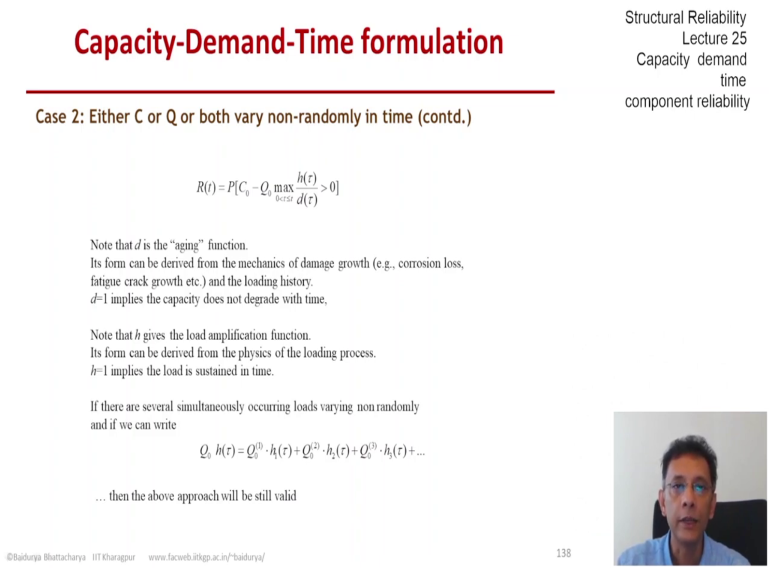


So, let us look at some more explanation of this. So, this d as we have said is the aging function if d is one there is obviously no aging is going on but in general it represents the effect of processes such as corrosion loss fatigue crack growth and so on. Likewise this function h is the load amplification factor if we may use that word and h equals 1 implies that the load is sustained in time which takes us back to case 1.

But in general h comes from our understanding of the loading process. Now what if we admit the possibility that there are several loading processes we have looked at things very simplistically. So, far as there are; only two quantities capacity and demand and they can be separated and we can look at the difference as a safety margin and so on. But what if there are more than one. So, let us say there is Q 0 1 Q 0 2 and so on.

All these different random variables and they are respectively multiplied by h 1 of tau and h 2 of tau which are each non-random functions of time. So, obviously it will be more difficult to take the ratio of h over d as we did above but we could bring it down to a two random variable problem if we can express the total load effect as one random variable times another non-random function h and then proceed as before.