

**Structural Reliability**  
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**Lecture –192**  
**Capacity Demand Time Component Reliability (Part 01)**

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**Structural reliability - course recap**

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- Part A
  - Motivation
  - Basics of probability
  - Basics of random variables
  - Common probability distributions
  - Joint distributions
  - Monte Carlo simulations - discrete continuous and dependent variate generation
- Part C (ongoing)
  - Capacity demand component reliability (time invariant)
    - First order reliability method
    - Second order reliability method
    - Monte Carlo simulations
    - Importance sampling

- Part B
  - History and scope of reliability studies
  - Definition and terminologies
  - Reliability problem formulation
  - System representation & redundancy
  - Time to failure based approach to reliability (phenomenological)
  - Random TTF, MTTF, hazard function
  - Estimation of TTF statistics from test data
  - Time dependent system reliability - phenomenological (TTF defined)

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With this lecture we start with capacity demand time formulation of component reliability problems. So, let us just take stock of how far we have reached in the course. So, in part A we looked at the basics the basics of probability, the basics of random variables and distributions. We discussed about the common distributions that appear quite regularly in structural reliability both discrete and continuous we looked at joint distributions and then we ended part a with a discussion on Monte Carlo simulations particularly the generation of discrete and continuous and dependent random variants.

In part B we started with the evolution of the subject of reliability defined the terms spent a good amount of time discussing how to formulate reliability problems and then how to describe systems in terms of its constituent elements, elements or components and discussed various types of redundancy that a system might have. Then we spent the rest of part B in taking the time to

failure based approach to reliability and we call it phenomenological meaning that the TTF is the only random variable that we have in either describing the reliability or the uncertainties in the aspects of the problem.

So, we looked at components and we looked at systems the system time to failure was defined in terms of the element times to failure and we discussed reliability functions hazard functions mean time to failure and so on. In part C; which is now ongoing we started looking at the physics based approach and the capacity demand time. We set aside time for the time being and we looked at just capacity demand component reliability time invariant.

And under that we have already seen the first order reliability method to compute the liability of the component the second order reliability method Monte Carlo simulations to compute failure probabilities and important sampling as one of the variance reduction techniques. So, today we bring in the time aspect explicitly in the capacity demand component reliability problem.

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## Recap: Reliability problem formulation

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### Phenomenological vs. Physics based

- Is the definition of satisfactory performance by the item:
  - Available in terms of the physics of the problem?
  - If yes, is the randomness in the physical variables known?
  - If yes, is their time-dependence relevant and known?

- Yes ⇒ Physics based reliability problem.
  - It is often called capacity demand reliability problem, or
  - stress-strength-time problem,
  - a special case of which is the structural reliability problem.
- No ⇒ Phenomenological reliability problem
  - Failure is identified by observation,
  - typically in term of time to failure (TTF)
  - which is the only available random quantity describing each component.



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So, let us just recap for a second how we defined this capacity demand physics based approach to reliability if we have available the definition of failure or the limit state in terms of the underlying physics in our case the mechanics of the problem and we know the randomness of the physical variables involved and particularly if there is any time dependence we know the

relevant time variation either random or deterministic.

And the answer is no to any of these questions obviously we go back to the phenomenological approach but here we have the answer and then that gives us the capacity demand time on this or the stress strength time approach to reliability and that is what we are going to look at today for a component.

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### Recall: Limit state and failure

$Rel(t, \Omega; \Gamma, \Theta)$  = Probability that an item occupying a logical or physical domain  $\Omega$  will perform its required function(s)  $\Gamma$  under given conditions  $\Theta$  for a specified time interval  $(0, t)$

The reference to the spatial dimension may be suppressed, and one defines failure as:

Failure =  $\{M(\tau) \in \bar{\Gamma}_{sp}\}$ , for any  $\tau \in (0, t)$

In capacity demand formulation,  
 $M(\tau) = f[C(\tau), D(\tau)]$ , in general  
 $= C(\tau) - D(\tau)$ , when separable

Consider one failure mode at one critical location  $\underline{x}$ :  
 Failure =  $\{M(\tau, \underline{x}) \in \bar{\Gamma}_{sp}\}$ , for any  $\tau \in (0, t)$ , for given  $\underline{x} \in \Omega$

A "component" is an item of reliability that describes one failure mode at one critical location.


Component limit state equation:  
 $g(\underline{X}; \tau) = 0$   
 such that  $g(\underline{X}; \tau) < 0 \Rightarrow$  failed  
 $g(\underline{X}; \tau) \geq 0 \Rightarrow$  safe

$C = C(\underline{X}_C; \tau), D = D(\underline{X}_D; \tau)$   
 It is not necessary that  $\underline{X}_C \cap \underline{X}_D = \emptyset$   
 Basic variables:  $\underline{X} = \underline{X}_C \cup \underline{X}_D$   
 $M(\tau) = g(\underline{X}; \tau)$

Probability of failure:  
 $P[\underline{X} \in \bar{\Gamma}_{sp} \text{ at any } \tau] = P(g(\underline{X}; \tau) < 0, \text{ any } \tau)$

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$g$  is variously known as performance function, safety margin, limit state function etc.



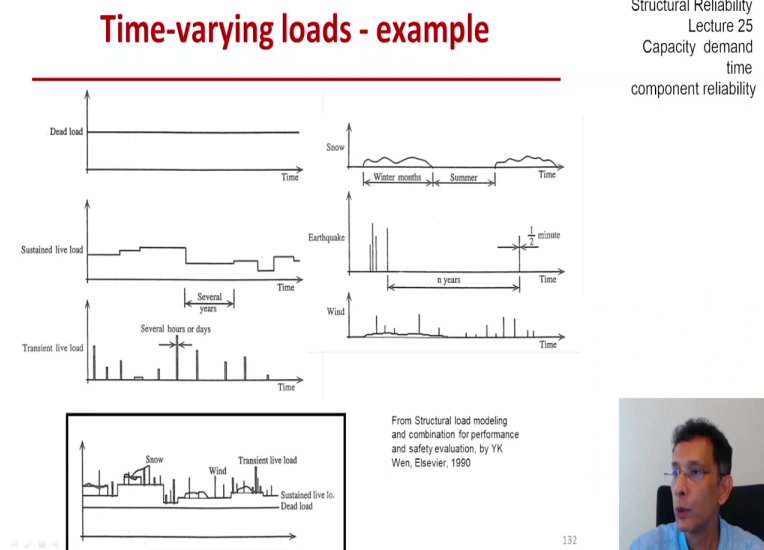
So, let us let us take a look at how we defined a component. In general we defined reliability as a probability that our item of interest would perform satisfactorily during the service life under given conditions and it must have some well-defined functions that it serves. This item occupies a logical or a physical domain omega. So, if we look at just one location or one identifier x, x does not have to be a physical location.

It is some identifier and then we can define failure at that location as some safety margin exceeding going out of the safe set during any time in the reference period. Now that is how we define component that the component is that item which can be defined with just one failure mode or one limit state. So, if that is. So, then we can for a component reliability problem we can suppress that reference to the location or that identifier and just simplify the definition of failure as the safety margin the time dependent safety margin going out of the safe set.

In capacity demand formulation this safety margin is a function of capacity type and demand type random variables if they are separable we could express  $m$  as the difference or the ratio  $C - D$  or  $C/D$  etc but these in turn are composed of or are functions of strength type and load type variables. We the whole set of those variables we call basic variables and in structure reliability that safety margin is called the limit state function the performance function and so on  $g$  which is a function of basic variables and time explicitly.

For the component if  $g$  is our limit state function  $g$  equals 0 is our limit state  $g$  less than zero implies failure  $g$  greater than or equal to zero implies safe for acceptable performance. And we define the probability of failure of that component in terms of that those random variables those basic variables are out of the safe set if we have the function the limit state function less than zero during any time from zero to  $t$  in the service life. So, let us see how this looks like through a very simple example for a component.

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So, here we have a cantilevered beam loaded by a sustained dead load  $w$  and a dynamic load  $p$  at the tip. We are interested in the tip deflection and want it to be limited to a certain value. So, we know the system we know its performance objectives and we have a very clear definition of what is the limit of satisfactory performance and you see the examples spelt out in words next to each

of them. Once we have defined our system completely we need to know which parameters which variables need to be considered.

So, that we are able to express through the physics of the problem through the mechanics of the problem the required response in terms of the system properties and the inputs to the system. We also need to know for each of those variables if they are random their entire probabilistic description and here the important point is that if there are time varying quantities like loads or degrading strains we need to have the knowledge of that in terms of the underlying mechanics.

So, with all of that we are able to create an input output model a finite element model for example and then we have the limit state in terms of the variables the mechanics and we also know the probabilistic information about all those variables and processes. So, that would let us compute the probability of failure. And we are particularly now interested in the time varying aspects of one or more of these quantities.

For example here you see the load this dynamic load  $p$  could be live load occurring at random instances of time with random magnitudes and they are added to the sustained dead load and the black lines are sample functions of degrading strengths and this degradation could be because of aging effects such as corrosion or fatigue crack growth. Let us take a look at some examples of time-varying loads taken from the book by Wen.

So, we have already seen the dead load under case the time invariant loads under case one. Live load could broadly be of two types the sustained type where any change that might occur would happen after several years. There could be transient live loads of the order of hours or days or even seconds and minutes. For example if we are talking about trucks on a bridge then each such heavy truck would be present only for a second or few seconds.

Natural hazards give rise to other kinds of loads with other kind of time variation snow loads earthquake loads wind loads and so on. And in the end they all come together and the total load

profile is the sum of all of these but not necessarily a linear combination there would be nonlinearities involved depending on the material behaviour depending on the geometric properties of our system.

So we need to combine them but we need to be careful how we combine them uh. So, that would give us the entire load random process and if there is any time variance would be captured that way.

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### Recap: Structural reliability – step by step plan


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3. Physics based component reliability formulation

1. – time-invariant case  $\Rightarrow$  Random C and D;
- ➔ 2. time dependent case  $\Rightarrow$  first passage problem

- Case 1. Both C and D are time invariant
- Case 2. Either C or D or both vary non-randomly in time
- Case 3. Load occurs as a pulsed sequence with random magnitudes
  - Known number of load pulses and no aging
  - Poisson load process and no aging
  - Poisson load process with deterministic aging
- Case 4. Both C and D are random and time-variant, but stationary in nature.



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So, this is the step-by-step plan we started with when we were discussing part B and part C of this course. We have already completed the first two steps the phenomenological approach to time to failure both for components and for systems and then in this part we have started the third step where we look at the physics of the problem. We have looked at the time invariant case which we called case one and now it is the turn of case two.

And later we are going to look at the system reliability formulation from again this phase physics based approach of capacity and demand. To elaborate on the cases that we are going to see in the time dependent situation we are going to look at cases two three and four in this and the next lecture. We have already seen case one where all the basic variables were time invariant today we are going to see at C or D or both capacity or demand both varying in time but non randomly

doing so.

And then we are going to look at a more interesting case where the loads occur as pulsed sequences with random magnitudes and random occurrence times as well. We will include aging there as well and in the end we are going to look at the more advanced case of both C and D varying randomly in time and then see what conclusions we can draw from that.