

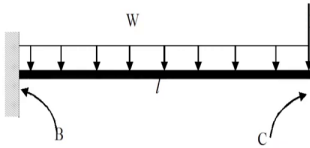
**Structural Reliability**  
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**Lecture –191**  
**Capacity Demand Component Reliability (Part 39)**

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**Importance sampling simulations**

Example E1 - cantilevered beam with deflection LS and 3 RVs



**Geometric properties**

$l=10$  ft

$I=98$  in<sup>4</sup>

**Failure criteria:** deflection based


Deflection at C:  $D=Wl^4/(8EI)+Pl^3/(3EI)$

**Limiting deflection:**  $d_{max}=1.5$  inch

RV	mean	c.o.v.	Distribution
P	3 kip	0.15	Normal
W	0.5 kip/ft	0.2	Normal
E	29000 ksi	0.05	Normal

all are mutually independent

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In the next problem with the important sampling we graduate from a one-dimensional cable in tension to a cantilever beam subjected to transverse loads. We have looked at this beam as an example structure many times but this is the first time we are going to solve this for a reliability estimation. So, we are calling this the example group E and the span let us say it is 10 feet the moment of inertia is 98 inch to the power 4.

The failure criteria we have given examples of this is deflection based and we are taking elastic analysis. So, the deflection at c which is the point of highest deflection is a linear contribution from w the distributed load and the point load P and you see the contribution of EI the flexural related in the denominator the limiting reflection is 1.5 inch. So, we need there are three random variables we need to define their statistics.

We take the loads to be random with the properties as you see on your screen and we just take E the elastic modulus as the random quantity we leave I the moment of inertia to be a non-random constant. So, with this in mind I would like you to solve this with an important sampling if you like you could solve it with form you could solve it with basic Monte Carlo but this would be a good exercise.

And I am going to give you the answers in the next slide and if you want to work through the example first then please pause and then look at the answer.

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graph TD
    A[Identify basic variables X] --> B[Obtain joint density function f_X]
    A --> C[Obtain LS eqn(s), g(X)=0]
    B --> D[Identify most probable failure region(s) in X space]
    C --> D
    D --> E[Propose IS joint density function f_i]
    E --> F[Start simulation]
    F --> G[Generate 1 ~ f_i]
    G --> H[Evaluate f_i(x)  
Evaluate f_j(x)  
Evaluate I_g(x) < 0  
Evaluate and store I_g(x) < 0 / f_i(x)]
    H --> I[Estimate Pf and associated uncertainties]
    H --> J[Repeat until complete]
    J --> F
        
```

### Importance sampling simulations

**Example E1 - cantilevered beam with deflection LS and 3 RVs**

IS distribution	mean	c.o.v.	Distribution
Pv	$\mu_P + 3\sigma_P$	0.15	Normal
Wv	$\mu_W + 3\sigma_W$	0.2	Normal
E <sub>v</sub>	$\mu_E - 3\sigma_E$	0.05	Normal
all are mutually independent			
means shifted		c.o.v.'s unchanged	

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So in this slide I am going to first give you the suggested important sampling density location it is not based on any FORM analysis or anything it is just arbitrary on the left you see the algorithm which we have already described and on the right in this yellow box I am suggesting that let us not change the distribution types let us keep the COV's also unchanged but let us just move the mean to for the load P 3 standard deviations above the mean for the load w also 3 standard deviations above the mean and for the strength type quantity E.

Let us move three standard deviations below the mean again if you want to do a FORM analysis which is recommended you could change these and perhaps you are going to find different answers and which would also be better because the FORM point after all is very close to the

maximum likelihood point.  
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### Importance sampling simulations

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Brute Force nmet	Monte Carlo pf	Importance nmet	Sampling pf	Importance Sampling nmet	Importance Sampling pf
10000000	0.000128	10000	0.000149	100000	0.000132
10000000	0.000132	10000	0.000126	100000	0.000133
10000000	0.000131	10000	0.000127	100000	0.000132
10000000	0.000134	10000	0.000119	100000	0.000136
10000000	0.000131	10000	0.000138	100000	0.000132
10000000	0.000133	10000	0.000129	100000	0.000134
10000000	0.000127	10000	0.000144	100000	0.000137
10000000	0.000133	10000	0.000149	100000	0.000132
10000000	0.00013	10000	0.000128	100000	0.000133
10000000	0.000134	10000	0.000136	100000	0.000133
10000000	0.000132	10000	0.000131	100000	0.000131
10000000	0.000134	10000	0.00013	100000	0.000137
10000000	0.000129	10000	0.000128	100000	0.000135
10000000	0.000131	10000	0.000122	100000	0.000134
10000000	0.000129	10000	0.000131	100000	0.000133
10000000	0.000139	10000	0.000144	100000	0.000136
10000000	0.000126	10000	0.000134	100000	0.000134
10000000	0.000133	10000	0.00014	100000	0.000126
10000000	0.000127	10000	0.000141	100000	0.000132
10000000	0.000133	10000	0.000143	100000	0.000135

ave	0.000131	0.000134	0.000133
sd	3.10E-06	8.72E-06	2.48E-06
c.o.v.	0.0236	0.0649	0.0186

RV	mean	c.o.v.	Distribution
Pv	$\mu + 3\sigma$	0.15	Normal
Vv	$\mu + 3\sigma$	0.2	Normal
E1	$\mu + 3\sigma$	0.05	Normal

all are mutually independent  
 means shifted c.o.v.'s unchanged

Example E1 - cantilevered beam with deflection LS and 3 RVs

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So, here are the results and what you see on the on the left two columns on the left group I have the result of 10 million Monte Carlo simulations basic Monte Carlo simulations brute force Monte Carlo simulations done repeatedly about I am guessing about 20 times and we have the answer from each of these. So, you see the answers vary in around something like 1 into 10 to the -4 and if we now look at an important sampling with what we are talking about.

We can get answers very close answers of the same order but only from 10000 important sampling simulations in each case and if we want to do a little more important something simulation then with 100000 each again we get answers that are very close. And now if we collate all these answers and get the overall statistics of all these individual runs this is the sort of answer that we are going to get.

The average of the brute force Monte Carlo with 10 million simulations is 1.31 in 10 to -4 that is the estimated Pf with the average and the COV is very small it is about 2.4% with just 10000 important sampling get an answer very close to the 10 million simulation answers from Monte Carlo basic Monte Carlo that is 1.3 into 10 to the -4. The uncertainty COV is 6.5% it is quite within our typical accepted limits a little higher than the two persons that we see from the basic

Monte Carlo but if we just did 10 times more number of simulations.

So, instead of 10000 we did 100000 we have an uncertainty less than 2% and obviously the answer the mean estimate is very close  $1.3 \times 10^{-4}$ . So, clearly here we are getting an advantage of almost two orders of magnitude the answer we get from 100000 the answer we get from 100 million I am sorry the answer we get from 10 million simulations we are getting only from 100000. So, there is a 100 fold increase in efficiency for kind of the same level of uncertainty.

I would encourage you to look at this problem solve it try out different important sampling location points see how FORM answers will affect your results and you can always compare with these results.