Structural Reliability

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Lecture –190
Capacity Demand Component Reliability (Part 38)

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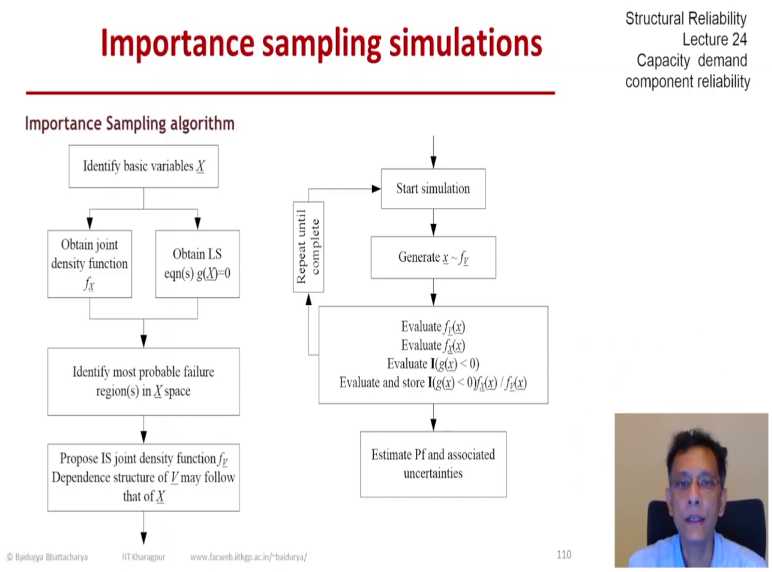

Let us now solve some problems with important sampling. Here is the algorithm that you see on your screen we have to identify the basic variables of the problem which we have been doing regularly the last few lecture. And then we have to know the joint density function of these basic variables and be able to define the failure criterion of the limited equation or equations when we say equations in the plural we mean there is a system level problem defined.

This algorithm is going to work equally well for systems but for now our focus is component reliability of the capacity demand type. So we are able to define the limit state and then an important sampling which was not necessary in basic Monte Carlo simulations is we have to identify or at least get an idea of the most probable failure regions made again in plural in the basic variable space.

Because that is where ultimately we want to do most of the simulation and then to do that once we know the most probable failure region we have to propose an important sampling density function. This density function we have been calling f v and if there is a dependent structure in f X f v may or may not have any dependent structure both are fine. If we want to have a dependent structure in f v as well then it may follow the same as that of f x.

Now once we are ready with these preliminaries we will start the simulation we will generate the basic variables from the important sampling distribution f v. So, it is the shifted the biased distribution and then with that X that we have sampled from f v we evaluate the density function f v there we evaluate the density function f X there the same point and then we evaluate if failure has occurred or not and if failure has occurred.

Then we store the entire quantity that we are taking the expectation of if you remember our basic setup that is f X over f v that ratio of the two density functions and i obviously is one if failure has occurred. So, we keep doing this until we are done with the simulations and whether we have reached convergence our uncertainty in the estimate is below our tolerance levels. We keep doing that and then once we are happy we come out of the loop and we estimate the Pf and the associated uncertainties and report those. So, this is the algorithm that we are going to follow.

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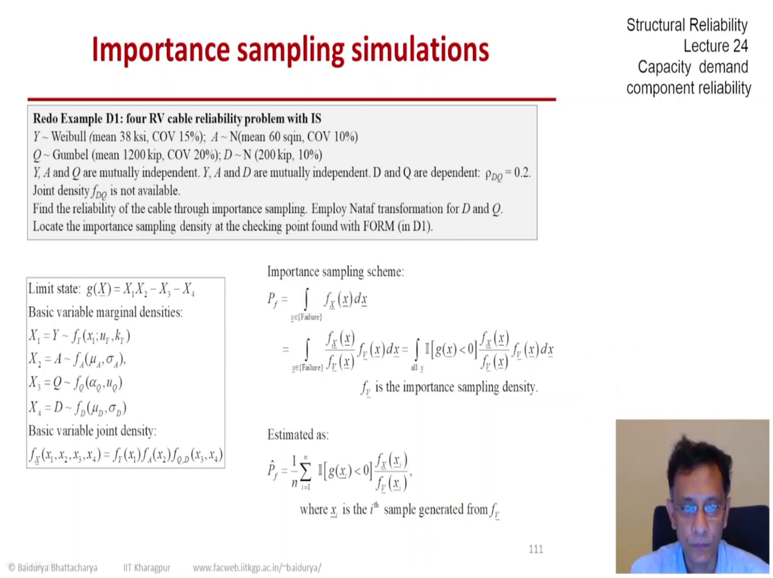

And the first example we take up is the one involving again our old friend the cable in tension it is the example group D, D 1 which has four random variables and let us just take a few seconds to read through the problem and then we will go through the steps in solving it. So, the nature of Y A Q and D they are the same as before there is a correlation coefficient between D and Q we have to employ Nataf transformation for them and we can use the most probable point found from form that we have already solved and there we are going to locate the important sampling density function.

So, this is our limit state it's X 1, X 2 - X 3 - X 4 we have seen this several times already and then the individual random variables are distributed as you see f Y f A f Q and f D for X 1, X 2, X 2 and X 4 respectively. The basic variable joint density function f X that is based on the information given it is the marginal density of Y times the marginal density of A times the joint density of Q and D and these are evaluated at X 1 X 2 and X 3 X 4 respectively.

The importance sampling scheme is we just for completeness we list it here the failure probability is estimated as the expectation of i f x over f v and these X's are generated from the density function f v and that is what we do repeatedly as I describe in the flow chart that the Pf estimate is the average of the important sampling multiplied by the density of the multiplied by the ratio of the two densities.

So, we now have to scheme of generating these four random variables from their important sampling density for x 1 and x 2 for yield and area it is straightforward because they are independent all we need is to be able to know their marginal distribution function and we can invert that with a uniform 0 1 deviate for Q and D we need to go through a few steps.

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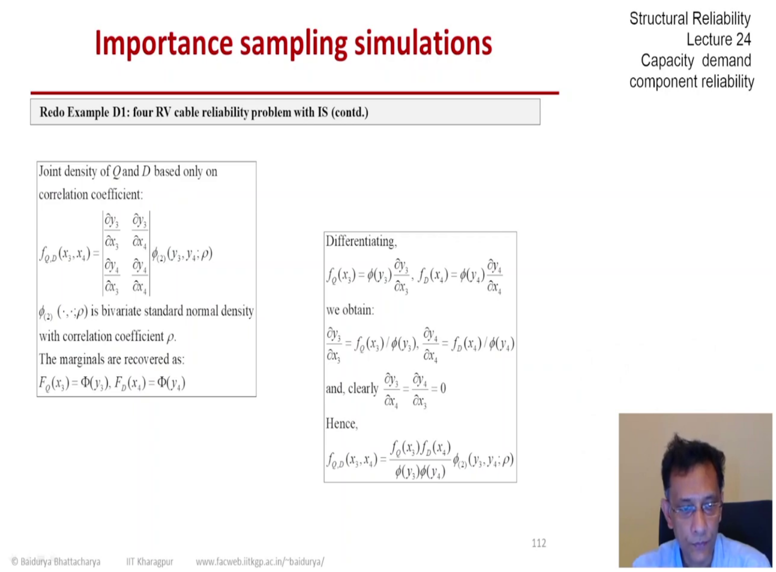

So, let us take them the joint density function of Q and D is not really defined all we know is that they are dependent and we have been given their correlation coefficient rho and we would like to we have done this before we would like to map them onto a bivariate normal density with the same correlation coefficient rho. So, if that is. So, we can transform Q D to Y 3 Y 4 Y 3 Y 4 being the standard correlated normal bivariate.

So, you see on your screen the Jacobian of the transformation and phi 2 is that bivariate normal density. So, we once we have the marginal once we have the joint density function we can obtain the marginal densities the marginal distributions and the marginals are recovered from the CDF to serial transformation now how do we get the density of Q and the density of D in terms of the marginal densities of Y 3 and Y 4 which are standard normal.

So we can use the chain rule of differentiation and partial Y 3 with respect to X 3 is the ratio of the two densities likewise for Y 4 and X 4. So, the jacobian we are almost ready to solve we recognize that the diagonal terms are 0. So, partial Y 3 partial X 4 and partial Y 4 partial X 3 each is 0 and that lets us write out the density function the joint density function of Q D, Q and D in terms of the bivariate normal density function multiplied by a correction factor which is the product of the marginal densities divided by the product of the marginal normal densities. So, this is something which we are going to use when we do the important sampling.

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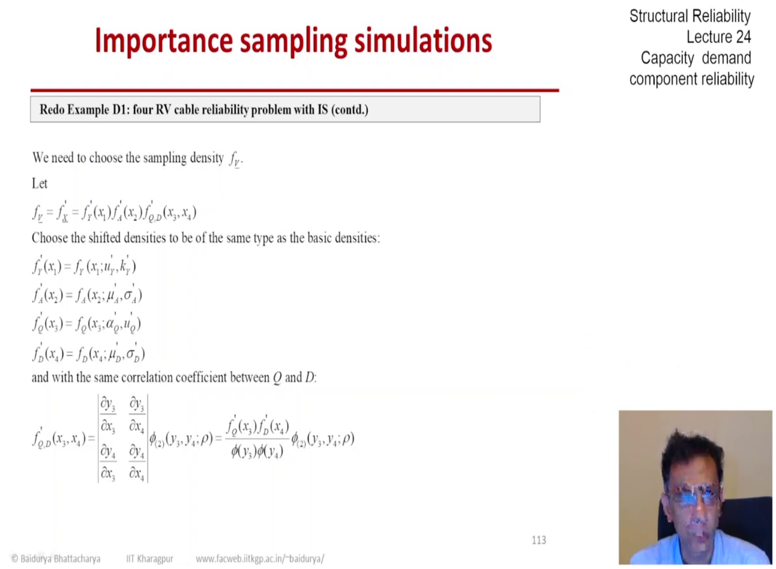

So, this is what the important sampling looks like the important sampling density function looks like we choose it to be this way that f v has the same form as f X and we expand it as f Y times f A they are both primed I hope you can see the prime in the superscript they are not specs of dust on your screen but they are supposed to be primes and f prime Q D for the dependent random variables. So, we choose these primed densities to be of the same form same type as the unprimed.

So, f prime of Y is the same type as f Y but with different parameters. So, the u and k of the viable distribution are potentially different. So, u prime and k prime likewise for the area f prime is the same type as f A prime the same type as f A but with different mean and standard deviation and same for Q and D for Q and D we would like to have the same correlation coefficient for f prime sub Q D and that tells us again using the same jacobian type transformation we used for the unprimed case is the joint density of Q D in the shifted sense.

The important sampling sense is again the product of the marginal primes f prime Q and f prime D over the marginal standard normals times the bivariate standard dot with the same correlation coefficient rho.

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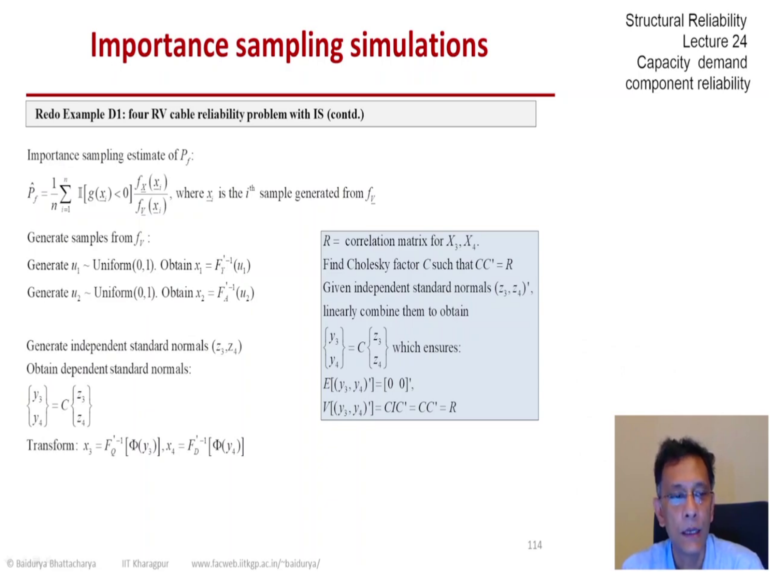

Now we are almost ready to start the simulation. So, this is our scheme. So, we now are ready to write out the ratio f X over f v and let us see what that looks like. So, as I said for yield and area if they are straightforward we have to just generate u 1 and invert that generate u 2 and invert that. So, that is not a problem for x 3 and x 4 for Q and D we have to bring in the dependence and we find the lower cholesky factor for R the correlation matrix and then with that factor c we can write the Y's Y 3 and Y 4 in terms of the independent standard normals z 3 and z 4.

And so, this is something we are just going to do once and then with that c we are going to define the density functions of Y 3 and Y 4 and hence X 3 and X 4. So, that is how we do Y 3 is a linear combination of Z 3 and Z 4, Y 4 is a linear combination of Z 3 and Z 4 that is how they are dependent and we individually transform them to get X 3 and X 4.

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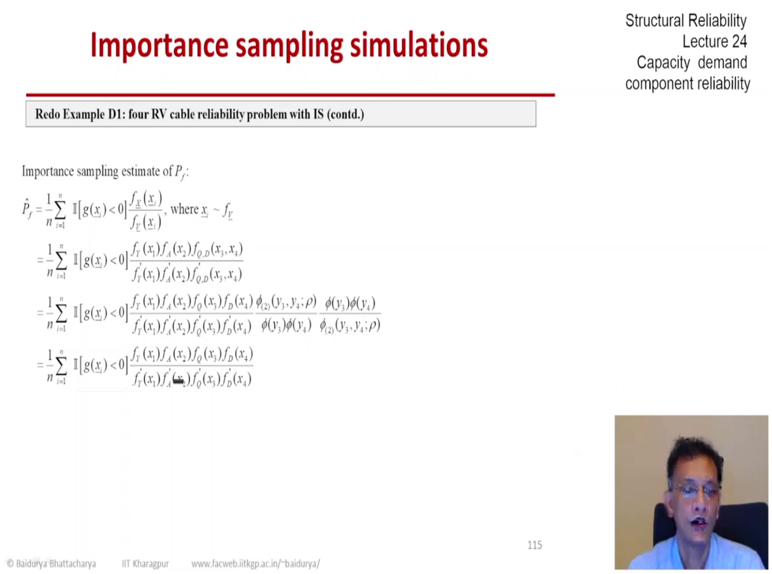

Finally we can now take care of the ratio of the two the two density functions and that is as you see on the screen on the on the numerator you have f X and on the denominator you have f v we have expanded them in the second line and using the Jacobians of the transformation because we have used the same row they cancel out and what we are left with is the product of the individual density functions the original individual density functions and in the denominator the product of the shifted density functions.

So, that is the correction that we have to use and if we do it repeatedly then that is how we would sum and divide by n to get the estimate of Pf. So that brings to an end the method and now let us look at the results.

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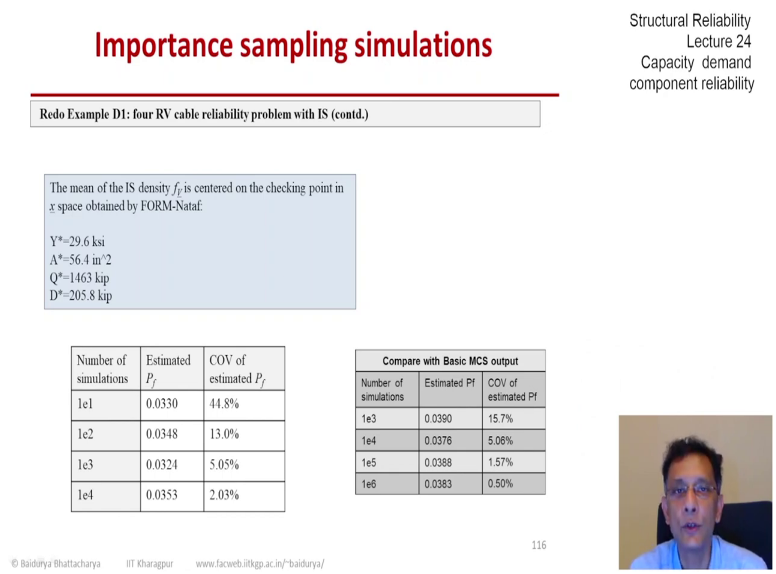

We have used this form net of results as we were asked. So that is our the mean of the important sampling density is centered on that form minimum distance point Y star, A star, Q star and D star as you see on your screen and then for the various number of simulations we have the estimated Pf and the uncertainty thereof and you see it is it is quite interesting and quite satisfying that even with 10 samples just 10 samples we are able to estimate the pf reasonably well it is something like you know 3.3%.

If we remember there's a huge uncertainty about 45% but we have not gone too off if we choose the density functions the important sampling density function that we did and the mean of f v centered on the form design point we are pretty good instead of 10 if we did 100 simulations we would stay around the same value of the estimate. But the uncertainty comes down significantly to 13. So, if our idea is to stay around 10 uncertainty maybe 100 or 200 simulations would be enough which is actually quite an improvement compared to the basic Monte Carlo simulations that we also have done for this problem.

And I am going to just repeat those results here for our for the sake of comparison but let us just see if we did a 1000 simulations the uncertainty comes down to about 5% and if we went overboard and did 10000 simulations the answer is like 2% uh. So, let us see where this stands in comparison with the basic Monte Carlo simulation output. So, to achieve a 15% uncertainty basic Monte Carlo would need about 1000 simulations but we achieved that in just a 100 an important sampling.

And if we wanted something like 1 or 2% we would need 100000 simulations which can be achieved in something like 10000. So, there is clearly a gain of one order of magnitude which could be huge when the computational demand is high and computational demand being high is not a thing of the past even though the computing power that we have now is orders of magnitude higher than when such methods were first developed in the 1970s and 1980s.

But the gap between the problems that we can solve and the problem that we want to solve that almost remains constant. So, the need for such variance reduction techniques is always there