Structural Reliability
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Lecture –189
Capacity Demand Component Reliability (Part 37)

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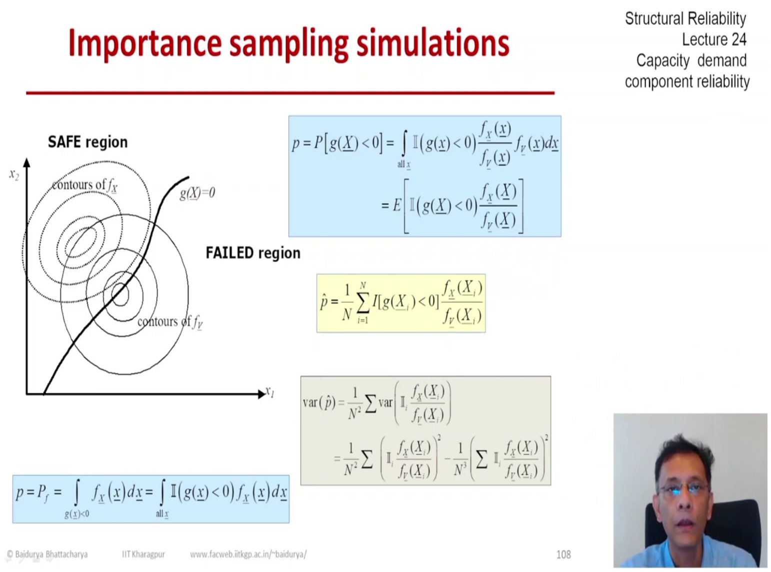

Now we present the method behind important sampling. Let us first put both of the densities the original density which we have been calling f(x) and then the modified density or the shifted density or the bias density that we saw in the movie in the previous slide land that is called the def v and let us put both of them on the same plane. So, as before we see the limit state function g the limit state equation g equals 0 and the failed region and the safe region have been demarcated.

And we see the two densities which are decidedly different not only are they centered on different regions but I have deliberately made the contours look different from one another because the f v density function does not have to mimic the density function of f x. So, this is our original scheme which we looked at quite a few times when we were looking at Monte Carlo simulations. So, our failure probability is the probability content of the failed region or g(x) less than 0.

And we can just convert that into an expectation type operation if we bring in the indicator function I which becomes zero when we are in the safe set and which becomes 1 when we are in the failed set. So, the domain of the density function now becomes the integration occurs over the entire domain of the density function f x. So, this by definition is the expectation of the indicator function capital I.

Now let us do something interesting and without changing of the definition or the expression for p at all, all we do is that within the integration we multiply and divide by the new density function f of v evaluated at the same point x. So, that is fine it is it is something we can definitely do but this gives us an idea and that is very important sampling comes in is this now could be interpreted as something entirely different it could be interpreted as the expectation of a larger expression.

The indicator function multiplied by the ratio of f x over f v and then the expectation in that case is being taken with respect to the density function f v no longer with density function f x. As long as we remember this there is no confusion but this is a completely different way of looking at the same problem now we are sampling from the new density function f v and the indicator function is multiplied by a sort of correction factor if you will f x over f v the ratio of the two density functions.

So, the summoned is zero when there is no failure but the summon is not exactly one when there is failure in fact it depends on x. So, that IID Bernoulli sequence and all the benefits that we could get from those properties they are lost. But still this gives us the way of estimating failure probability which is what we did actually in the previous example. So p hat the estimated failure probability would be the average of the expanded quantity i times the ratio of f x over f v.

And the sampling is being done from the density function of v f v. Then obviously we could try to find out what the uncertainty in the estimate is and the uncertainty in the estimate would be the sum of the individual variances. The variance in p hat would be the sum of the individual variances; because the samples we are getting from f v are still IID it is just not from f x but still there are IIDs. So, we can express the variance of the sum as sum of the various multiplied by the coefficient squared which is 1 over n squared.

In this case and if we obtain these n samples we could express the variance of p hat simply as expectation of x squared minus the square of the expectation. So, the last line that you see is basically that. So, this is our new estimate of the failure probability and depending on how we choose f v this can become quite efficient. Unfortunately if we choose f v wrongly then we would actually have widely incorrect results.

So important sampling has to be done carefully the location on which you center the important sampling density f v is vitally important let us look at an example to see what the result of choosing a very special sampling density function would look like.

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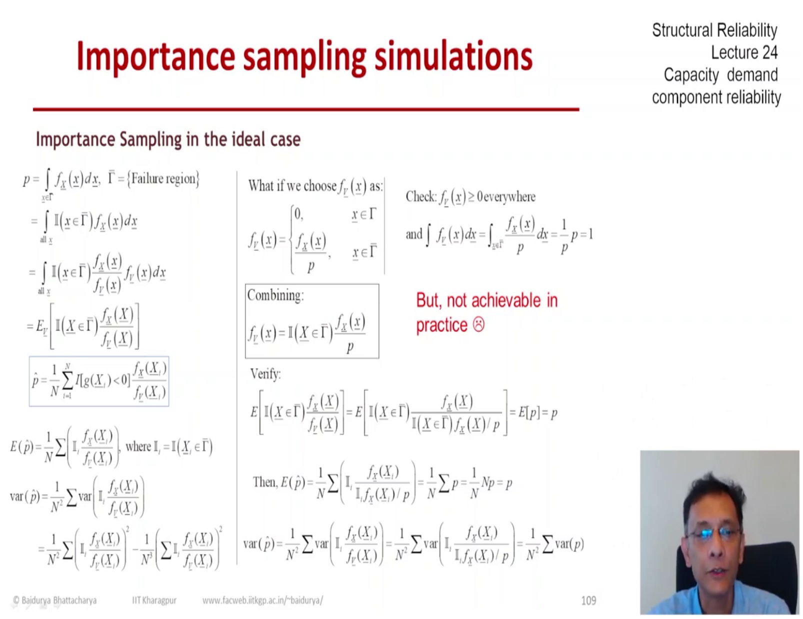

So, this is the ideal case and we can show that in this ideal case the important sampling will have actually 0 variance and in fact just one just one sample would give me the correct answer. Let us see how we can create that special sampling density. So, this is where we are our definition our expression for the failure probability is the expectation as you see on the screen and we estimate that way and the mean and the variance of the estimate is as you see on the screen.

So, this is all we had in the previous slide but we need to use them. So, that is why I put them here on the left and now let us see what if we choose the sampling density f v as 0 in the safe region and the original density divided by the content in the failed region the probability content. So, that is p at that in the failed region. So, and in principle I can do this very well because I know the original density function f of x that is what I start with and I can always truncate a density ignore one part and then enhance the remaining part amplify it so, that the volume or the area under that still equals one.

So, let us check if that is what is happening. So, obviously f v this way defined is non-negative everywhere. So, that is good and if we integrate over all possible values of x we still get the value of 1. So, f v is indeed a legitimate density function we also need to; let us combine the two conditions that x belongs to gamma the safe set and x belongs to gamma bar the failed set and we can we can write the f of v the sampling density combined as the original density function times the indicator function that x is in the unsafe set divided by p.

So, that is one combined expression. Now let us just make sure that this way the expected value of i f x by f v indeed gives us p. So, let us just go through the steps if we put the value of f v as we have defined just above then indeed we see that the expectation boils down to expectation of p which p being a constant is p itself. So, this way we actually are being true to the original setup of the problem.

Now let us see in this situation the way we have defined f v what is the expected value of p hat the expected value of p hat if we go through the steps just plug in the value of f v in the equation on the left side it gives us 1 over n times the sum of p which basically is p itself. So, we are back to an unbiased estimator. So, that is great and what would be the variance of p hat if f v is i f x over p that turns out to be something very interesting is if you just plug in the values it is 1 over n square times the sum of the variances of p but p is a constant.

So, the variance of a constant is 0. So, the variance of p hat is zero. So, which basically means p hat is a constant and just one estimate of the sample is good enough for me to estimate p exactly. The problem obviously is as you have noted is that we do not know what p is our whole idea is to be able to estimate p. So, even if this is the ideal situation we can really not achieve it because we need prior knowledge of something that we do not know.