Structural Reliability
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Lecture –188
Capacity Demand Component Reliability (Part 36)

We continue with part C of our course in this part on capacity demand type problems in reliability we have so, far looked at FORM the First Order Reliability Method the SORM the Second Order Reliability Method and in the previous lecture we looked at Monte Carlo simulations for component reliability problems. Today we are going to look at importance sampling as one of the variance reduction techniques in Monte Carlo simulations. Now why would we be interested in something like this.

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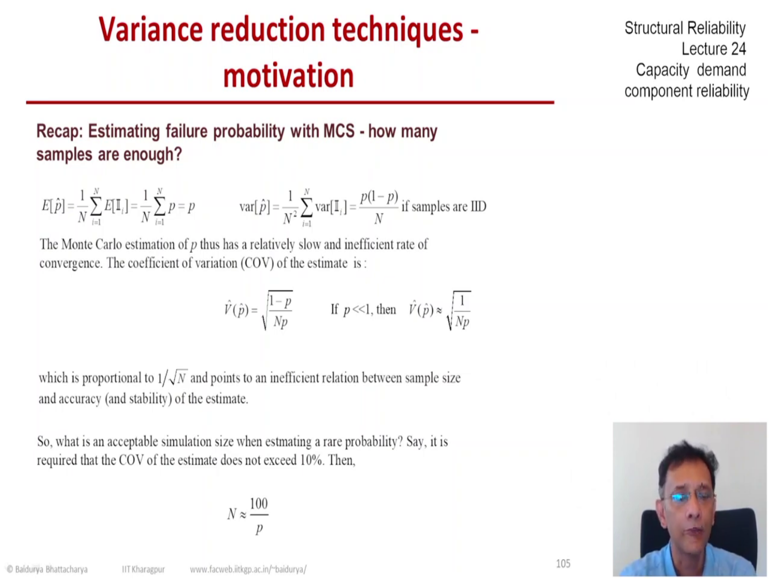

Let us recall what we learned yesterday we perform Monte Carlo simulations repeatedly to estimate an unknown probability typically a very small probability by generating iid samples from the joint distribution of the basic variables of the problem. And the estimate p hat which is in some sense an average of an indicator function uh is uh a random quantity because of the finite sample size. So this estimate p hat has a mean of p itself.

So, it is unbiased its mean is the true value its variance the variance of p hat goes down with one over n provided the samples are IID which actually points to an inefficient convergence it is the cov the coefficient of variation of the estimate goes down with 1 over square root of n. So, the question naturally is that can we can we do better here is an example that we saw if we want to limit the cov to 10% our number of samples that we would need to generate would be of the order of 100 over p.

So, just as an example if p is say one in a million which is very small but suppose it was we would need to generate about 100 million samples or 100 million trials which could be very demanding if every trial required a detailed and involved finite element analysis of the structure. So, whether it is one in a million or one in 100000 you get the idea. So, let us look at this uh this issue of uh inefficient convergence and what might be done about it uh in a graphical manner.

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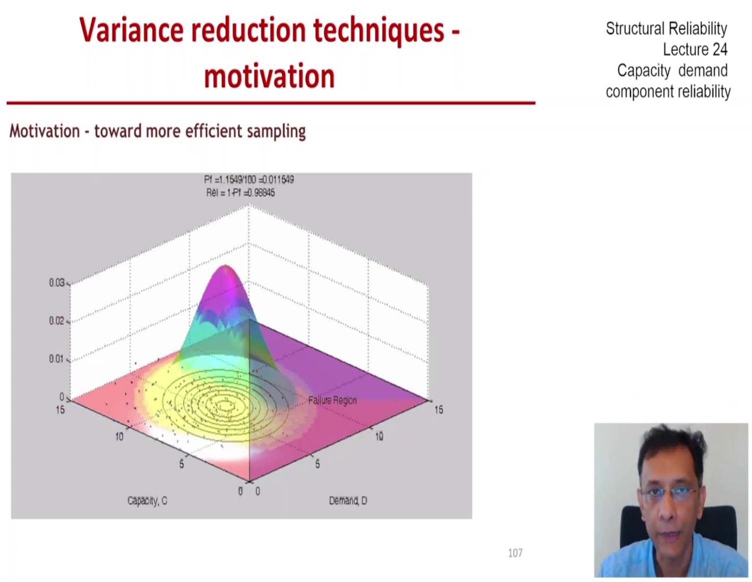

So, let us see if I can run this movie this is a by varied problem we have capacity c and d and the limit state is a straight line between C and D it is C equals D and we will generate just 1000 samples and estimate P f. So, what you see are the concentric circles on the plane which indicate the pdf the joint pdf contours of C and D and the third dimension is the probability density. So, clearly and as is quite usual most of the probability mass is concentrated well into the safe area.

So, it is quite obvious that we are going to get a point generated on the plane quite rarely in the failure region and that is exactly what happens as we see in this movie. So, we have started generating points and you can clearly see that most of these points have fallen in the safe region and it appears something like about 11 points out of 1000 fell in the failed region.

Let us run the program again let us run the movie again and see; what we are getting. So, this time I am going to pause it just before the end. So, it is something like 11 in a 1000. So, one might wonder that. So, many points that fell in the safe region are in some sense wasted we find use only for those 11 points or so, that gave rise to failure. So, can we not do something can we either make use of those points that fell in the safe region or alternatively can we generate more points in the failure region.

Now obviously that is not the true situation but perhaps we could correct for that bias and let us see what would happen if we did something like that. So, here we have this um more efficient sampling sort of scheme let us let us run the movie we are going to just generate 100 points and let us see what comes out of that. The first thing you will see that we are moving the density function closer to the limit state closer to the failure region.

So, let us see if the movie works yes. So, we move the density function closer to the limit state and now obviously many more points have fallen in the failure region and then it seems that our estimated reliability is still about 1.15% which agrees quite well with the basic Monte Carlo scheme that we used in the previous slide there also we have about 11000 about 1.1% . Let us run this movie again uh just to see what is happening and then we will continue with the formal presentation of this method.