

**Structural Reliability**  
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**Lecture –187**  
**Capacity Demand Component Reliability (Part 35)**

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**Monte Carlo simulations**

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**Recap: Example D1: four RV cable reliability problem**

We solve the earlier cable reliability problem (D1) with MCS.  
 $Y \sim$  Weibull (mean 38 ksi, COV 15%);  $A \sim N$ (mean 60 sqin, COV 10%)  
 $Q \sim$  Gumbel (mean 1200 kip, COV 20%);  $D \sim N$  (200 kip, 10%)  
 $Y, A$  and  $Q$  are mutually independent.  $Y, A$  and  $D$  are mutually independent.  $D$  and  $Q$  are dependent:  $\rho_{DQ} = 0.2$ .  
 Joint density  $f_{DQ}$  is not available.  
 Find the reliability of the cable using MCS. Employ Nataf transformation for  $D$  and  $Q$ .

$X_1 = Y(u_1, k_1)$ $X_2 = A(u_2, \sigma_A)$ $X_3 = Q(u_3, u_4)$ $X_4 = D(u_5, \sigma_D)$  $g(X) = X_1 X_2 - X_3 - X_4$  Generate random deviates $X_i, i=1,2$ (each independent of all others) $x_i = F_i^{-1}(u_i)$ where $u_i \sim \text{Uniform}(0,1)$ $F_i =$ CDF of $X_i, F_i^{-1}$ is the inverse CDF	Generate random deviates $X_i, i=3,4$ (mutually dependent variables, independent of $X_1, X_2$ ): $R =$ correlation matrix for $X_3, X_4$ . Find Cholesky factor $C$ such that $CC^T = R$ Generate independent standard normals $(z_3, z_4)$ , linearly combine them to obtain dependent standard normals $(y_3, y_4)^T$ : $\begin{Bmatrix} y_3 \\ y_4 \end{Bmatrix} = C \begin{Bmatrix} z_3 \\ z_4 \end{Bmatrix}$ , which ensures $E[(y_3, y_4)^T] = [0 \ 0]^T, P[(y_3, y_4)^T] = C C^T = CC^T = R$ Transform: $x_3 = F_3^{-1}[\Phi(y_3)]$ , $x_4 = F_4^{-1}[\Phi(y_4)]$
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The last problem we solve in this lecture is our old friend the cable reliability problem the four variable version of that which we called example D1. So, let us take a few seconds to recap this problem statement and then we will solve this using Monte Carlo simulations we have already solved this with FORM and SORM. So, we have four random variables  $X_1, X_2, X_3, X_4$  out of which  $X_1$  and  $X_2$  are each independent of the other three and  $X_3$  and  $X_4$  are dependent on each other. So, our limit state function looks like  $X_1 X_2$  minus  $X_3$  minus  $X_4$ .

And in Monte Carlo simulations we have to generate these  $X_1, X_2, X_3, X_4$  repeatedly. So, for the first two for  $X_1$  and  $X_2$  it is trivial we all we would need is to use the inversion method  $X_1$  is the yield strength viable  $X_2$  is the area normal. So, we have the ability to generate them from standard uniform deviates by the inversion method. So, that is straightforward we already did this in the previous example involving the save stopping distance.

But for the correlated ones we have to make sure that we ensure that the dependent structure is maintained. So,  $X_3$  and  $X_4$  are dependent on each other but they are independent of  $X_1$  and  $X_2$ . So, I will let  $R$  be the correlation matrix of  $X_3 X_4$  that is all we know about their dependent structure we do not know the joint distribution. So, we have the correlation matrix the 2 by 2 matrix and we do a cholesky factor the lower Cholesky factor  $C$  such that  $C C^T$  is equal to  $R$ .

And then we generate independent standard normals  $z_3$  and  $z_4$  and then we combine them linearly to obtain dependent standard normals  $y_3$  and  $y_4$  with the help of  $c$ . So,  $y_3 y_4$  are linear combinations of  $z_3 z_4$  and just to make sure that this way we get the mean of  $y_3 y_4$  as zero and the  $V$  of  $y_3 y_4$  is  $r$  and then we do an inversion of  $y_3$  to  $X_3$  and  $y_4$  to  $X_4$ . This way we ensure dependence between  $X_3$  and  $X_4$  however because of the non-linear transformation between the  $X$ 's and  $y$ 's we do not end up with the same  $R$  that we started with.

We have discussed this before if it is very important to ensure the exact value of row between  $X_3$  and  $X_4$  we should start with different values that we did at the top while finding the cholesky factor but even without that the error is very slight in most cases.

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## Monte Carlo simulations

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Redo Example D1: four RV cable reliability problem with MCS (contd.)

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nmct=1000000; %total number of MCT trials
count=0; %initialized failure counter

for i=1:nmct
    Y=u*V*(log(1/rand))^(1/kY);
    A=muA+randn*sda;

    z(1)=randn;
    z(2)=randn;
    y=cholCoor*Q*z'; %generating correlated y(1) and y(2)

    Q=muQ-(1/alphaQ)*log(-log(normcdf(y(1))));
    D=muD+y(2)*sdb; %transforming y(1) and y(2) to Q and D respectively

    if (Y*A-Q-D<0) %checking for failure
        count=count+1;
    end
end

pf=count/nmct
covpf=sqrt((1-pf)/nmct/pf)

```

Number of simulations	Estimated Pf	COV of estimated Pf
1e3	0.0390	15.7%
1e4	0.0376	5.06%
1e5	0.0388	1.57%
1e6	0.0383	0.50%



So now we are ready to present the MATLAB code and then the solution. So, we start with defining the basic variable statistics we do it for Q then we do it for A we do it for D and then we introduce the we bring in the correlation structure between Q and D. So, we define the 2 by 2 correlation matrix and then take the Cholesky factor and the last command in that line that you see is to make sure that it is the lower.

So, risky factor then we obtain the statistics of y. So, because y is viable we go through that step we have shown those before to get the shape and scale factors of the viable distribution then we are ready to start the loop we have 1 million trials once again here we are going to change that and show how the results keep getting better we initialize the count. We start the FOR loop and then we generate the random variables one by one we first generate y from the command rand which gives us the standard uniform deviate.

And we invert that to get the variable y we use the rand n command to generate the standard normal deviate and then transform that to the normal A and then for Q and D we generate two standard normals independent standard normals z 1 and z 2 and then we linearly combine them to get y using the lower choice Q factor and then once we have y one and y 2 we individually invert them to get Q and D.

So, Q comes from y 1 and d comes from y 2 using the full distribution transformation and now we are ready to check for failure. So, our limit state is  $y - A - Q - D$ . So, if failure happens then we increment the failure count by 1 and that brings us to the end of the for loop and we are now we can output the estimated failure probability and the cov of that estimate we have run this for four values of NMCT the first one is just 1000 estimates and that gives about 3.9% failure probability but the cov is a bit too large about 16%.

So, let us increase the number of samples the next one is 10 000 still about 3.8% the cov comes down to 5%. So, we could stop there but let's see how the improvement goes on uh. So, 10 to the power 5 samples gives us about 3.9% and 1.6% cov and then if we do a million simulations it is

still about 3.8% and it is half a percent of error. So, we can stop now we could have stopped much earlier and this result of a 3.8% we actually used when we were comparing the accuracy of FORM and SORM.