Structural Reliability
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Lecture –183
Capacity Demand Component Reliability (Part 31)

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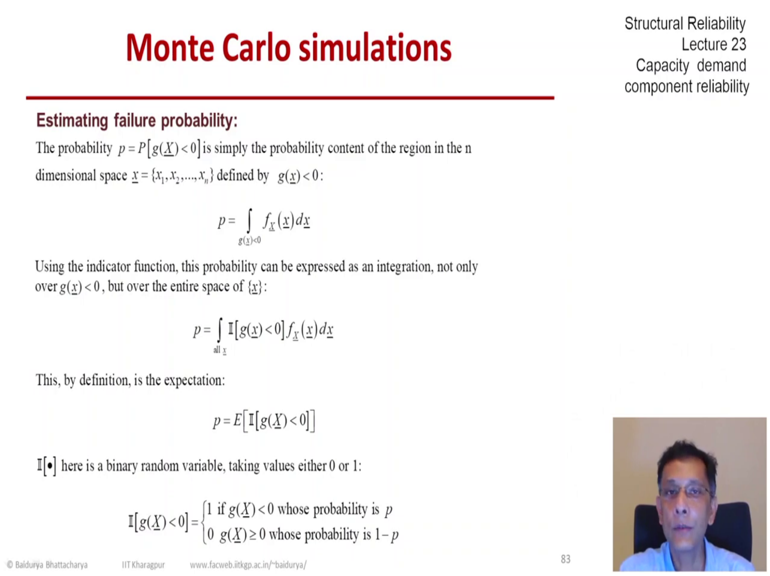

So how can Monte Carlo simulations help us in estimating failure probabilities? We remember that Monte Carlo simulations is basically sampling random numbers sampling random deviates from various distributions we will come back to that later in this lecture but let us make sure we understand that how multi-colour simulation comes in into this structural reliability problem. So, this we had in the previous slide that the probability small p the failure probability is the probability that the limit state function is less than zero.

So, this is an n dimensional integration of the joint density function of all the x's over the region of interest the failure ratio. So, this much is clear. Now we can make a clever substitution and instead of integrating over the only the failure region we can integrate over the entire domain of the problem. So, we will not restrict the region over we perform the integration. So, we will integrate over all x and to do that the trick is to introduce the indicator function.

So, this indicator function I that is scripted i that you see whose argument is g(x) less than 0 basically it is a binary function it is a truth function. So, and we have seen this function earlier in this course also. So, it takes on the value one when the argument is true and zero when the argument is false. So, in effect we did not change the problem statement at all but now we have the integral performed over the entire domain of the problem.

So, this lets us basically identify this problem statement as an expectation because by definition expectation is the integration of a random variable or its function weighted by the density function of that random variable and integrated over the entire region. So this then by definition is the expectation of the indicator function. So, my failure probability is nothing but the expectation of the indicator function identifying failure that is actually very useful it is not only interesting to see it that way but it is also going to be very useful.

Because this i is a binary random variable it takes on 2 values 0 and 1 and we are very familiar with the binary or the Bernoulli random variable and its probability of taking on value 1 is p and then taking value 0 is 1 - p and not only that we can we can find its mean its variance and so on.

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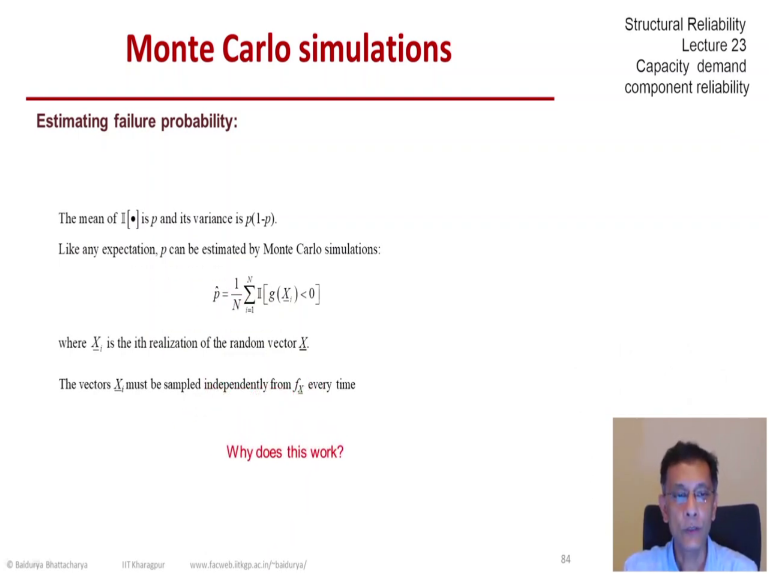

So, how do we use this we use this as a repeated sampling from x computing the value of g every time see if it is less than zero or not if it is less than 0 then the indicator function is one and we sum the indicator function and divide by n. So, that in effect is computing an expectation that's how we estimate expectations from Monte Carlo simulations or any sampling method we sample the random variable or function that we want to find expectation of and then if we do it enough number of times we divide by the number of samples and we get an estimate of the mean or the expectation.

Obviously it is a good idea to use random samples IID samples then we can be sure that the variance is going to keep going down and eventually we are going to reach the right answer and why that is. So, that is actually backed by a very fundamental law of probability. So, but the point to note here is that for that to take effect for that fundamental law to be able to come into force we must make sure that these x's are sampled independently every time from the joint density function of x.

Now obviously within the vector x there can be dependence we are not talking about that we are talking about the vector being sampled from its distribution every time those samples must be mutually independent.

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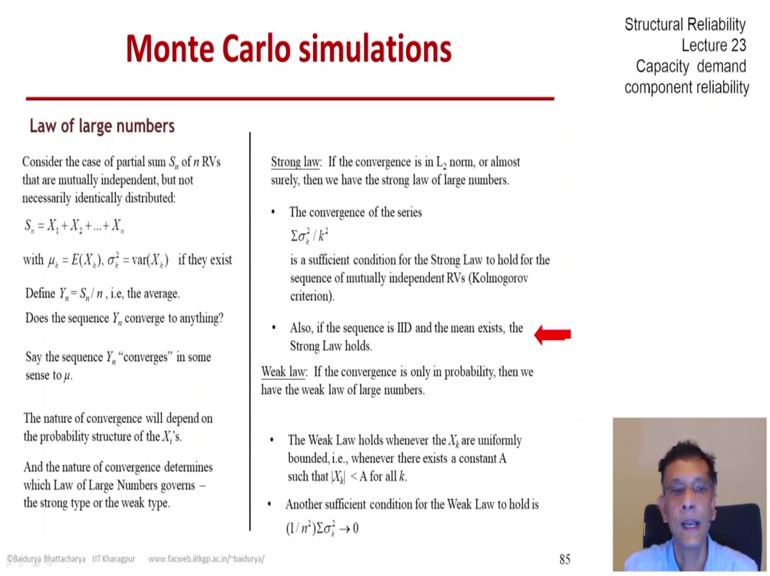

So, then why does it work it works because we invoke the we can invoke the law of large numbers and this is something we discussed when we were discussing joint distributions again in part A of this course if you wish please go back to those lectures and those slides. Just basically to summarize we have the sum of a large number of independent random variables not necessarily identically distributed but what does the average converge to.

And depending on the nature of the underlying random variables the probabilistic nature the convergence can be in weak form or strong form and it so, happens that between the strong law and the large the strong law and the weak law for large numbers if we have IID samples a sequence of IID random variables then the strong law holds. So, it is in our interest if we can make sure that the samples are IID from which we are estimating the expectation of a function of those samples.

Then we can get the backing of the strong law of large numbers and that must converge to the true failure probability. So, that is the strength behind this approach.