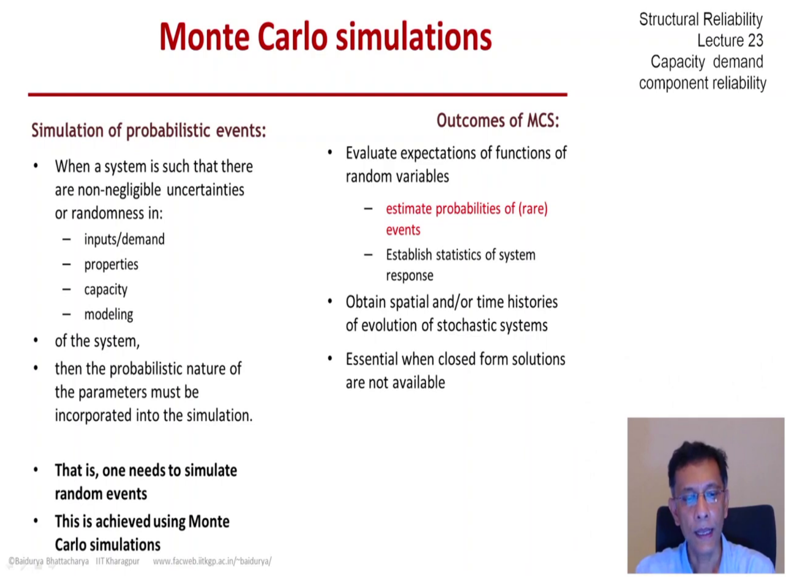
Structural Reliability

Prof. Baidurya Bhattacharya
Department of Civil Engineering
Indian Institute of Technology, Kharagpur

Lecture –182
Capacity Demand Component Reliability (Part 30)

(Refer Slide Time: 00:43)

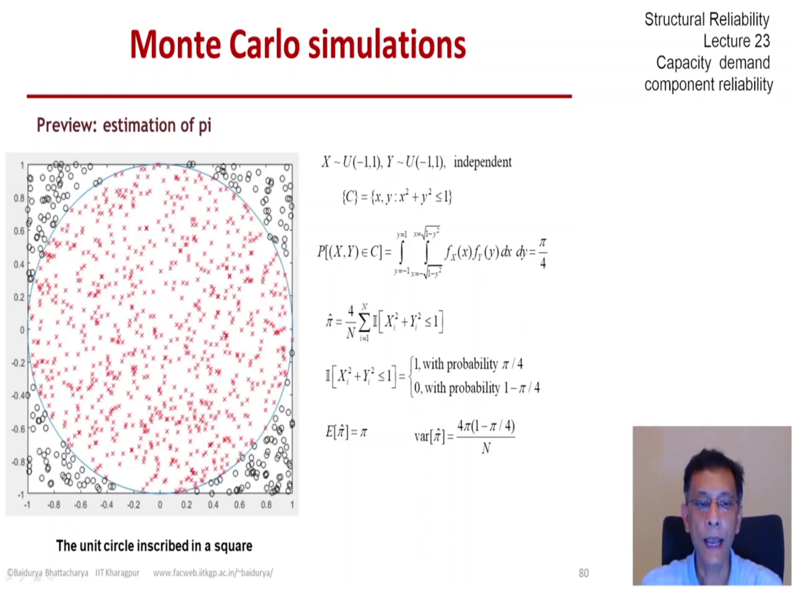


Monte Carlo simulations for solving structural reliability problems, earlier in this course in week three of part A we had a brief introduction to the subject and this is what we concluded. If we are simulating a system and that system has non-negligible uncertainties in one or more of its aspects then we need to incorporate that probabilistic information in the system simulation. And that can be done those simulation of random events is done through Monte Carlo simulations.

The outcomes of MCS are many types we can evaluate expectations of functions of random variables which in turn can be used for estimation of probabilities especially rare probabilities or establish some system statistics some response statistics or we can obtain time histories of a system response. And this is obviously essential when we do not have a closed form solution of the system's behaviour.

What is important and what is most relevant here when we are talking about MCS for structural reliability problems is how to use it for estimating rare probabilities and that is what I have highlighted in red on your screen. So, that is going to be our focus in today's lecture.

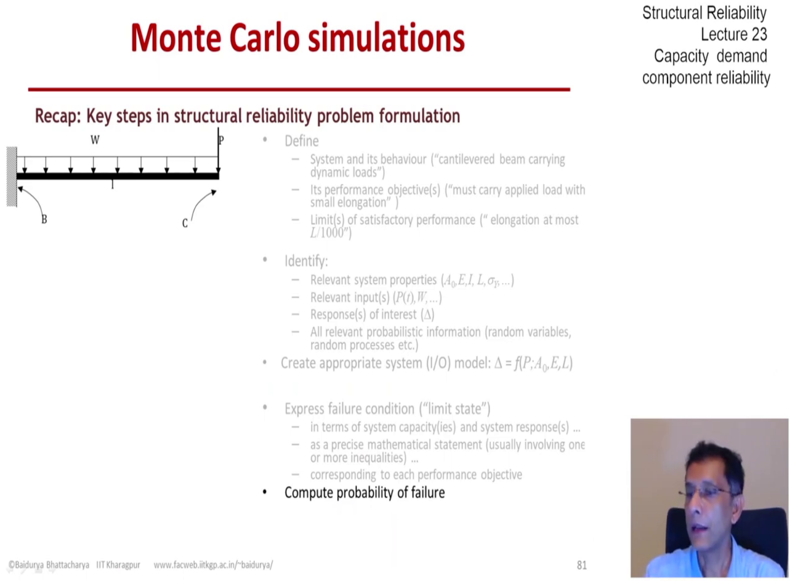
(Refer Slide Time: 02:22)



Now we solved one example problem during that first introduction which was estimation of I and we showed that pi can be estimated using a probability it was not a rare probability of course but it can be done very elegantly by simulating random points random points in the unit are circle inscribed in a square of size 2 by 2. So, intuitively and then it can be proved that if the points are indeed generated randomly in that square then the fraction of points falling within the circle would be related to the ratio of the two areas the area of the circle to the area of the square.

And that gave us the probability of generating points within the circle as pi by 4 and we showed that if we estimate this by Monte Carlo simulations if we can generate x and y independently between -1 and +1 and from a uniform distribution each then we can indeed estimate pi and the estimate became better and better as more and more samples were generated. So, this when we look at in the structural context, what you see here the estimate is a random variable its mean is the true value and its variance is inversely proportional to the sample size. So, these concepts can be carried nicely to a structural reliability situation.

(Refer Slide Time: 04:22)

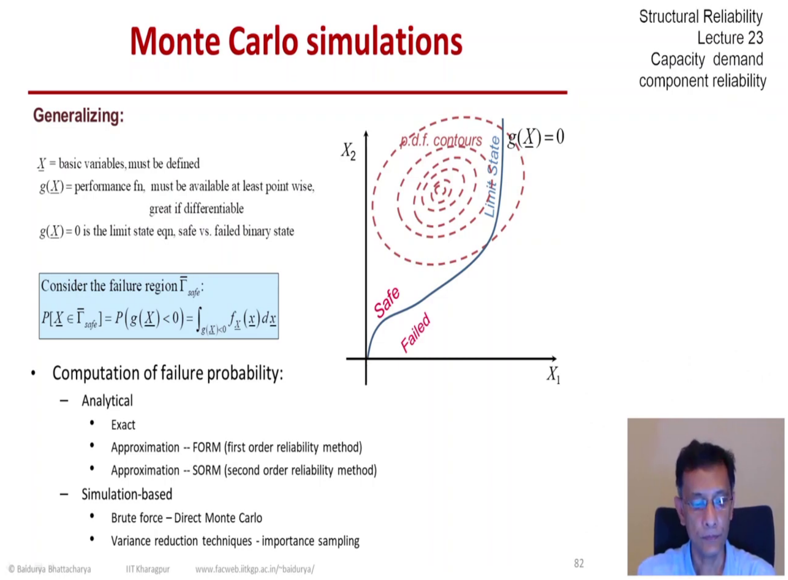


So, let us let us recap what we want to do in formulating and solving a structural reliability problem. So, here let us say we have a beam under two loads and we have a good definition of the system and its performance and what are the clear limits of satisfactory performance. We have been doing this for the last few weeks and we can identify the properties that are important in the mechanics of the problem.

And any uncertainty any relevant probabilistic information we should have or we should be able to get and then we have an appropriate system model which relates the output with the inputs and the system properties. And obviously we are looking at physics based definition of system behavior and failure which is why we are talking about this capacity demand type reliability. And now once we have been able to express failure in a clear mathematical term involving the mechanics of the problem and the system parameters.

Then we are able to or we should like to compute the probability of failure and we did this the last few lectures using an approximate method called FORM first order reliability method and an improvement to that called the second order reliability method.

(Refer Slide Time: 05:55)



So, if we generalize this in the context of the random variables x what we have is we have a set of basic variables which define the problem a performance function and this part of the course we are now talking only about a single performance function we are talking about element or component level reliability. So, but this can easily be generalized especially in the context of Monte Carlo simulations.

So, we have a performance function g of the x vector and g of x equals 0 is our limit state equation and on the right you see on the two dimensional space of x 1 and x 2 how this g of x can split the domain into safe and failed or unsafe regions. So, when we compute failure probability what we essentially do is estimate the probability content of that failed subset. So, this is then the mathematical description of the probability of failure.

We have the failure region gamma safe complement and the probability content of that region that x belongs to a gamma safe complement is by definition p of g x less than 0 and that is given by the n dimensional integration of the joint density function of the x vector over the region where g is less than 0 or the unsafe region or the failed region. So, this has been our problem set up and as I said we look broadly at two kinds of methods of computing that failure probability simple problems can be solved analytically.

And we solve many of them and then we started looking at approximate methods the FORM the SORM and today we are going to look at simulation based methods the direct Monte Carlo also known as Brute Force Monte Carlo. And then in the next lecture we are going to look at important sampling which is one of the variance reduction techniques.