

**Structural Reliability**  
**Prof. Baidurya Bhattacharya**  
**Department of Civil Engineering**  
**Indian Institute of Technology, Kharagpur**

**Lecture –181**  
**Capacity Demand Component Reliability (Part 29)**

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**Capacity Demand example with SORM**

Structural Reliability  
 Lecture 22  
 Capacity demand  
 component reliability

**Recap: Example D1: four RV cable reliability problem**

We now add a second load  $D$  (dead load) on the cable and consider  $D$  and  $Q$  to be statistically dependent.  $Y, A$  and  $Q$  continue to be mutually independent, as are  $Y, A$  and  $D$ .

$Y$  ~ Weibull (mean 38 ksi, COV 15%);  $A$  ~  $N$ (mean 60 sqin, COV 10%)  
 $Q$  ~ Gumbel (mean 1200 kip, COV 20%);  $D$  ~  $N$  (200 kip, 10%);  $\rho_{DQ} = 0.2$ . Joint density  $f_{DQ}$  is not available.

Find the reliability of the cable with SORM. Employ Nataf transformation.

$X_1 = Y(\mu_Y, k_Y), X_2 = A(\mu_A, \sigma_A), X_3 = Q(\sigma_Q, \mu_Q), X_4 = D(\mu_D, \sigma_D)$

$g(\underline{X}) = X_1 X_2 - X_3 - X_4$

$\min_{\underline{u}} \underline{u}^T \underline{u}$   
 such that  $h(\underline{u}) = 0$

$\underline{u}^* = -1.39, -0.607, 1.13, .067$   
 $\beta = 1.893$

Principal curvatures of approximate paraboloid:  $5.437e-5, -0.07289, -0.1449$

Paraboloid after rotation:  $\hat{n}_1 = 1.893 + 5.437 \times 10^{-5} \hat{n}_1^2 - .07289 \hat{n}_1^2 - .1449 \hat{n}_1^2$


$$p_{r,2} \approx \Phi(-\beta) \prod_{i=2}^{n-1} \frac{1}{[1 + \beta \kappa_i]^{1/2}}$$

$$= \Phi(-1.893) \frac{1}{[1 + 1.893 \times 5.437 \times 10^{-5}]^{1/2}} \times \frac{1}{[1 - 1.893 \times .07289]^{1/2}}$$

$$\times \frac{1}{[1 - 1.893 \times .1449]^{1/2}}$$

$= 0.3689$

$\Rightarrow \beta_{SORM} = \Phi^{-1}(1 - 0.3689) = 1.788$       Beta from MCS = 1.770



Problem D1 we solved this with FORM before now we are going to solve this with SORM and see what sort of improvement we might be getting. Let us take a few seconds to read the problem statement and then we will present the solution. So, this one is a four random variable problem and there is dependence between  $D$  and  $Q$  the two loads. Now we discussed in detail how we employed the native transformation how we solved it using the MATLAB function `f main con`.

So, we are not going to go into that we will just present the FORM results obtained from this Nataf transformation and then from that point onward we are going to derive the corrections necessary for SORM if you would like to solve this yourself please pause the video and otherwise let me present the answer step by step. So, this was our limit state  $X_1 X_2$  minus  $X_3$  minus  $X_4$  and we minimized the problem in the distance in the independent standard normal space.

And obtained a  $u^*$  as you see on the screen negative 1.39 negative 0.6 1.13 and 0.07 and the beta was about 1.89. So, that is our starting point and again after doing the eigen analysis we are something equivalent we get this curvature which for the first one is almost zero and the two other ones they are both negative. So, that gives us an equation of the paraboloid in terms of  $u_1$   $u_2$   $u_3$  giving drawing a surface above the  $u_4$  axis if you will.

And this gives us the three principle curvatures that we can use to get the correction factors and that are what we see the value of 5 of negative 1.893 obtained from FORM corrected by three factors. The first one is actually slightly less than one but the other two the second and the third ones are greater than one. So, we look like we are going to have a higher failure probability when we employ the SORM corrections and that is the answer 0.036 and if we find the equivalent beta by inverting the normal cdf we get a beta in SORM and equivalent beta of 1.788 instead of 1.893.

So, again this time SORM reduce the reliability estimate increase the failure probability and let us see if it is closer to the accurate value again accurate in the sense it is from our Monte Carlo estimates but we have been saying that if the sampling is done properly and if we do a large enough number of samples then the by the law of large numbers we should be getting closer and closer to the true value from MCS.

So, let us see how close or not this is well it turns out to be it is rather close the beta from MCS is about 1.77 the beta from SORM is about 1.79 whereas the beta from FORM is 1.89. So, in all these examples that we have been seeing SORM does seem to be making a good correction to the errors in FORM. Now obviously there are many issues including how many curvatures there are in the surface in the limit state surface if there is a point of inflection or not all these things would tell us what sort of or what quality of correction SORM is doing.

If there are multiple local minima all these things are a little advanced in nature and beyond the

scope of this course but I have shared some of the papers for further reading and you will be able to get more information from them.