

Structural Reliability
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Lecture –180
Capacity Demand Component Reliability (Part 28)

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Capacity Demand example with SORM

Structural Reliability
 Lecture 22
 Capacity demand
 component reliability

Recap: Example C1: three RV cable reliability problem
 We now consider Y, A and Q to be random, and mutually independent. The properties are as follows:
 The yield strength $Y \sim$ Weibull (mean 38 ksi, COV 15%)
 The cross-sectional area $A \sim N$ (mean 60 sqin, COV 10%)
 The load $Q \sim$ Gumbel (mean 1200 kip, COV 20%)
 Find the reliability of the cable with SORM. Employ full distribution transformation.

$$X_1 = Y(u_Y, k_Y), X_2 = A(\mu_A, \sigma_A), X_3 = Q(\alpha_Q, u_Q)$$

$$g(\underline{X}) = X_1 X_2 - X_3$$

$$\min_{\underline{u}} \underline{u}' \underline{u}$$

$$\text{such that } h(\underline{u}) = 0$$

$$\underline{u}^* = -1.622, -0.6535, 1.427$$

$$\beta = 2.257$$

Principal curvatures of approximate paraboloid: -0.1564, -0.04552

Paraboloid after rotation: $\hat{u}_i = 2.257 - .1564\hat{u}_1^2 - .04552\hat{u}_2^2$

$$p_{r,i} \approx \Phi(-\beta) \prod_{i=1}^n \frac{1}{[1 + \beta \kappa_i]^2}$$

$$= \Phi(-2.257) \frac{1}{[1 - 2.257 \times .1564]^2} \times \frac{1}{[1 - 2.257 \times .04552]^2}$$

$$= .01575$$

$$\Rightarrow \beta_{SORM} = \Phi^{-1}(1 - .0158) = 2.151$$

Beta from MCS= 2.149



We next look at problem C1. So, this was a three random variable problem and we already solved this with FORM. We will now solve it with SORM and let us take a few seconds just to read the problem statement. There are three random variables Y, A and Q and their distributions are given and we employed the full distribution transformation when we solve with FORM and that is what we will do we will solve the first few steps exactly as in FORM.

So, X_1, X_2 and X_3 are defined as before the limit state is $X_1 X_2$ minus X_3 and we minimize the distance in the independent standard normal space \underline{u} subject to h equals 0 h being the mapped limit state function and we got the answer which is \underline{u}^* as you see on the screen negative 1.62 negative 0.65 and 1.43 and the reliability index was about 2.25, 2.26. So, with this starting point let us see how much improvement we get if we employ SORM.

So, the principal curvatures of this approximate paraboloid in the remaining 2 coordinates are as you see negative 0.1564 and negative 0.04542 and how does our parabola look like see if you can imagine in three dimensions there is a curved surface in u_1 and u_2 u_1 tilde and u_2 tilde and u_3 is the third dimension. So, that is what our parabola looks like in this rotated coordinate system and we can now use these curvature values this negative 0.156 for a negative 0.04552 and correct the value obtained from 4.

So, that turns out to be again we have factors that increase the value of P_f because we discounted some of the we did not count some of the failure regions when we looked at FORM. So, the answer turns out to be 0.01575 and if we take the inverse normal cdf we get an equivalent beta of about 2.15 compare that with beta of 2.26 obtained from form. So, we see that there is a significant amount of improvement or change from first order to second order analysis but obviously which one is correct?

We did solve this problem with MCS with Monte Carlo simulations and the answer was an equivalent beta of about 2.15. So, clearly in this example also an approximate second order correction almost removes all the error and we end up with an answer that is almost the same as the accurate the exact value.