Structural Reliability
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Lecture –18
Review of Random Variables (Part - 01)

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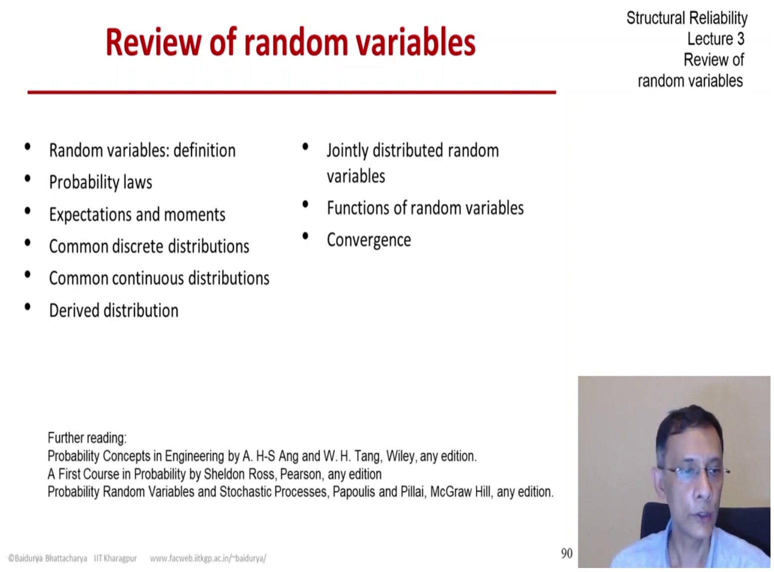

We are going to review random variables in this and the next 3 lectures. I would like to cover the definition, the probability laws that govern random variables, expectations and moments the common discrete and continuous distributions that we will encounter in this course and how they arise and their interrelations, change of variables, joint distributions, functions of random variables and the convergence of a sequence of random variables.

In addition to the recommended text by Ang and Tang I would also like to recommend the book by Sheldon Ross and the book by Papulis and Pillai for further reading.

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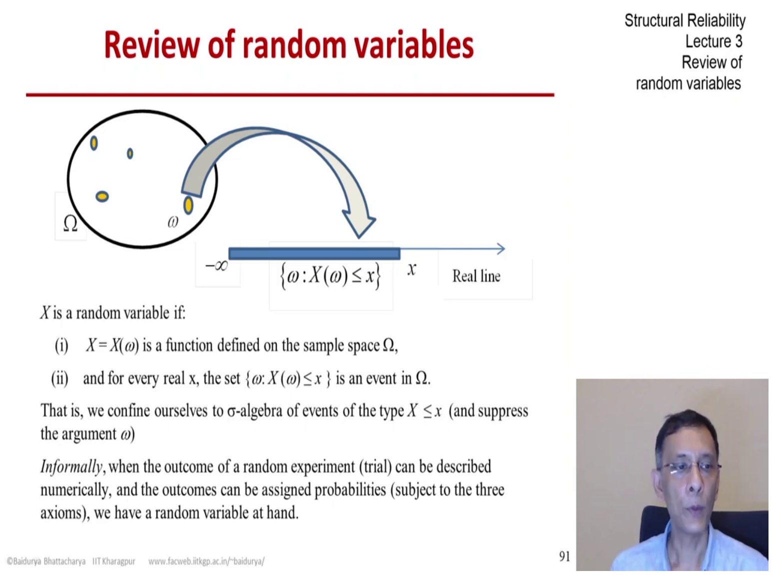

Now a random variable is a measurable function defined on the sample space. So, it maps events from the sample space on to intervals that you see on the screen and we discussed measurable functions in the previous lecture. Informally when the possible outcomes of an experiment or trial can be given in numerical terms and they can be assigned probabilities obeying the actions of probability then we have a random variable in hand.

So, when an experiment is performed the outcome of the random variable is called a realization a random variable can be discrete or it can be continuous and as I said it is governed by its probability laws. If a quantity varies randomly with time we model it as a stochastic process. A stochastic process can be viewed as a family of jointly distributed random variables if a quantity varies randomly in space we model it as a random field which can be thought of as a generalization of a stochastic process in two or more dimensions.

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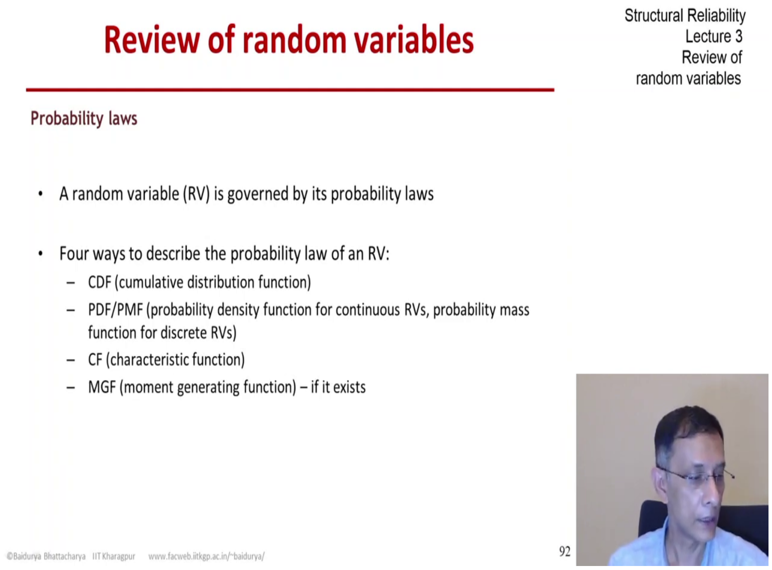

Now the probability laws of a random variable there are 4 ways to describe it the cumulative distribution function or the CDF, the probability density of the probability mass function depending on whether the random variable is continuous or discrete in nature. The characteristic function and the moment generating function if it exists.

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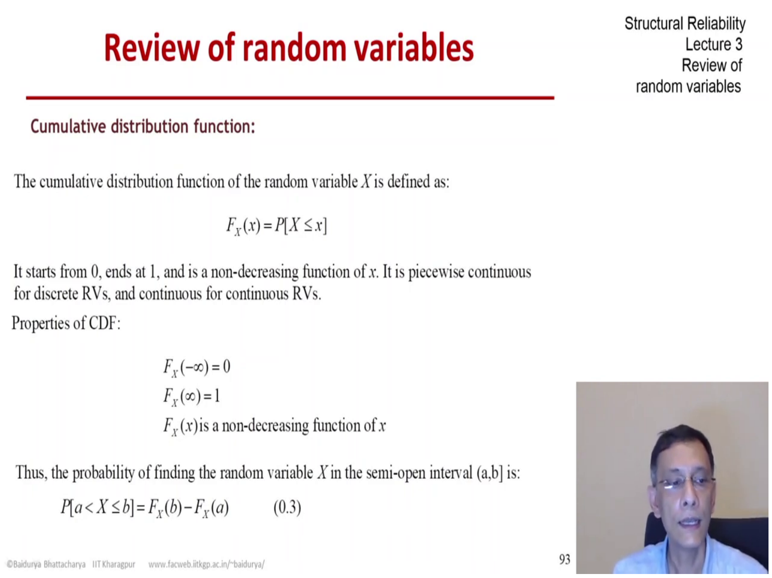

Now the CDF is the probability that our random variable assumes values less than or equal to little x. Our convention is to use uppercase letters to denote random variables like X, Y, Z and lowercase letters like a b c and little x little y little z to denote its realizations. So, the CDF it starts from zero ends at one and it is a non-decreasing function. If you want to find the probability that the random variable X takes on values between a and b strictly greater than a and less than or equal to b it is the difference of the CDF's evaluated at b and a.

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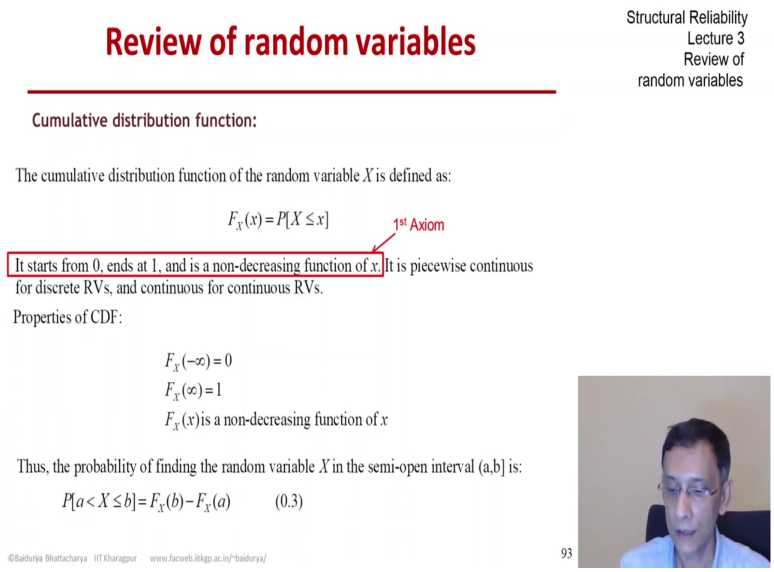

And if you would like to find out which of these corresponds to which actions of probabilities. So, this statement corresponds to the first action. This statement corresponds to the second action and the last is the equivalent of the third action.

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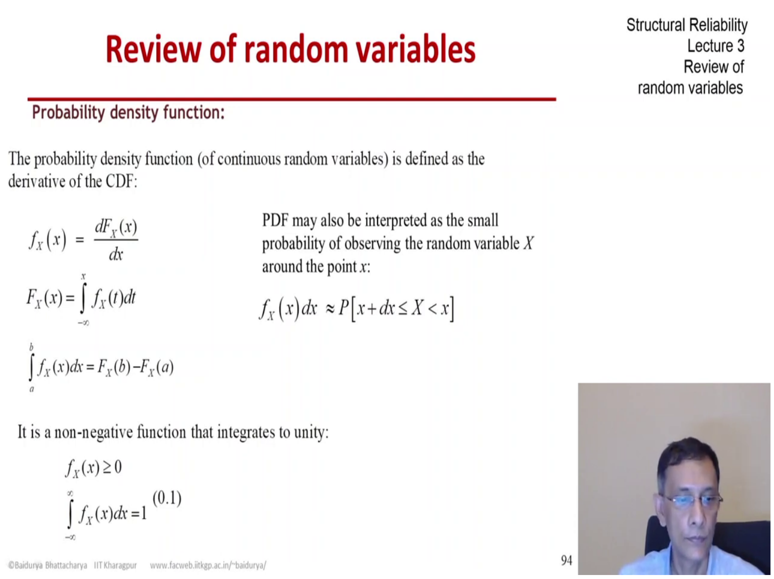

The density function is the derivative of the CDF or equivalently the CDF is the integration of the density function the area under the density curve is the probability that the random variable takes on values between the limits as I said. You can also think of the density function as it gives a measure of the random variable taking on values around that particular x. So, it is Fxdx is a small probability its properties are it is a non-negative function that integrates to unity.

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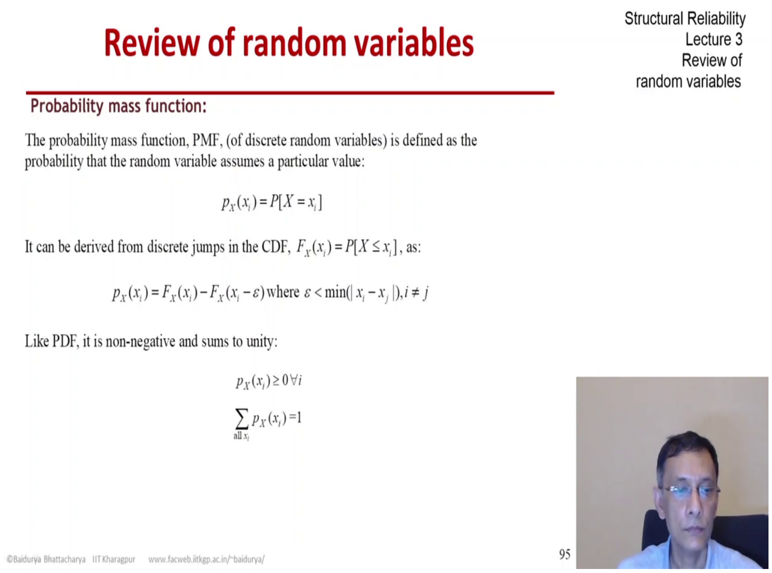

For discrete random variables we have the mass function. So, it is the probability that the random variable takes on that particular value x i lowercase x i you can also identify the PMF from discrete jumps in the CDF as you see here and if you sum all the PMF's these are all non-negative numbers. If you sum all the PMF's you get the value of 1.

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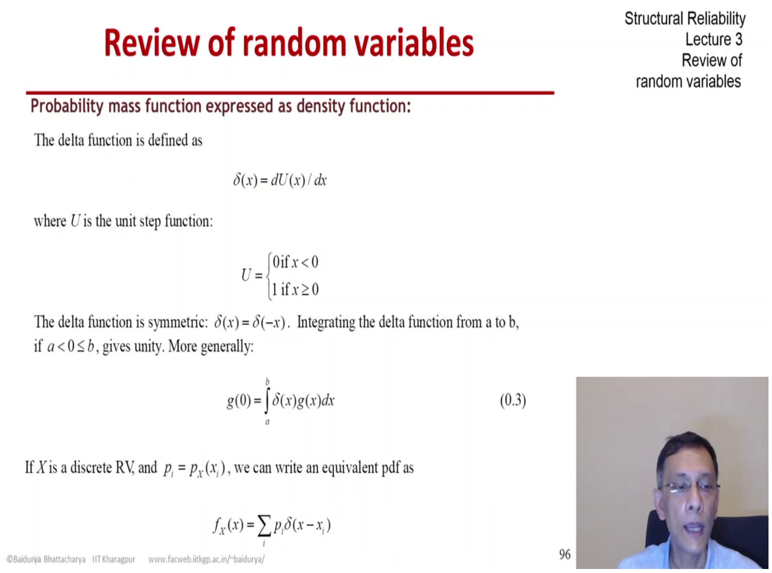

It is possible to describe mass functions as density functions by the help of the delta function. So, as you know the definitions of the delta function as I have put on the screen. So, you can describe the density function equivalently as the sum of the individual PMF's multiplied by the delta functions as you see on your screen.

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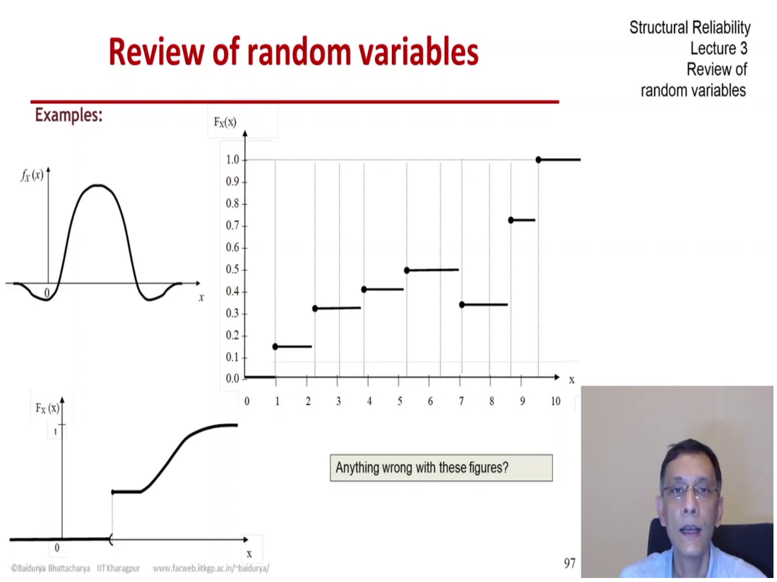

So, I have I am going to put 3 figures here and let us see if you can identify if there is anything wrong with these. So, the first one is a density function the second one is a CDF of discrete random variables and the third one is also another kind of CDF. So, the first one is actually not right the density function can never be negative. The second one also has a problem because the CDF can never go down as you can see at 7 it goes down.

And the last one it is fine it is non-decreasing. So, it is okay and this is a particular case of a mixed distribution. So, this is where sometimes the random variable takes discrete values and in other cases it takes continuous values.