

Structural Reliability
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Lecture –179
Capacity Demand Component Reliability (Part 27)

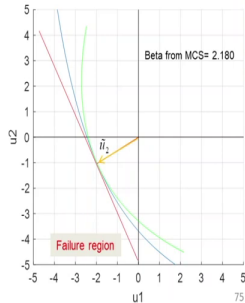
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Capacity Demand example with SORM

Structural Reliability
 Lecture 22
 Capacity demand
 component reliability

Recap: Example B4: two RV cable reliability problem
 We continue with the yield strength Y and cross-sectional area A as random variables and the axial load q as deterministic.
 $Y \sim N$ (mean 38 ksi, COV 15%), $A \sim N$ (50 sqm, COV 10%). Y and A are independent. $q = 1200$ kip.
 Find the reliability index with SORM.

Choose $X_1 = Y, X_2 = A$ Choose $T: u_i = (x_i - \mu) / \sigma$
 $\therefore g(\mathbf{X}) = X_1 X_2 - q$
 $h(\mathbf{u}) = 28.5u_1\mu_2 + 285u_1 + 190u_2 + 700$
 FORM optimal point: $u_1 = -1.97, u_2 = -1.03, \beta = 2.225$
 Principal curvature of approximate paraboloid: -0.08187
 Paraboloid after rotation: $\tilde{u}_1 = 2.225 - 0.08187\tilde{u}_1^2$
 $p_{j,2} \approx \Phi(-\beta) \prod_{i=1}^{j-1} \frac{1}{[1 + \beta\kappa_i]^2} = \Phi(-2.225) \frac{1}{[1 - 2.225 \times 0.08187]^2}$
 $= 1.303e-2 \times 1.106 = 1.440e-2$
 $\Rightarrow \beta_{SORM} = \Phi^{-1}(1 - 1.440e-2) = 2.186$



We will now solve some of the same problems we solved earlier with FORM but this time with SORM the Second Order Reliability Method. So let us just spend a few seconds looking at the problem statement this is problem B4 so there are two random variables Y and A the load Q is deterministic. So the first few steps in SORM are the same as those in FORM. So we write the limited equation and let us say we choose the Hasofer-Lind transformation between X and u so this gives us a limit state equation h is a limited function in the independent standard normal space u .

And this is what the limit state looks like pictorially so we have u_1 and u_2 and the blue curve is the limit state the origin is in the safe set and the failure region is towards the lower left. Now we did solve this problem with FORM and the optimal point was negative 1.97 for u_1 and negative 1.03 for u_2 and the beta the minimum distance was 2.25. So we had reached up to this point

when we are looked at FORM.

This red line was the approximate linear function that we fitted at the minimum distance point and the probability of failure or the reliability corresponding to this straight line when dealing with FORM. So now we can find we can employ the additional steps in SORM. So we rotate the coordinate system along the direction of that red line from the origin to the minimum distance point the alpha as the direction cosines and that gives us the direction of u_2 tilde.

So once we have u_2 tilde defined in that tilde axis system we can define the parabola so in two dimensions this is what it looks like u_2 tilde is the minimum distance minus a quadratic term so that would be our fitted parabola second order parabola at that line at that point. And this is what it looks like so the green line is the fitted parabola the second order approximation. And now let us use the formula that we already shared we have the approximate value of P_f as the FORM value corrected by a set of terms in product.

So the we just have one term there because it is a two dimensional problem and our problem size there is up to n minus one so the main curvature we already found that is point negative 0.08187 that is there on your screen and now we just apply the formula systematically. So we have this time a correction factor which is more than one, one divided by the square root of a number which is less than one so that turns out to be 1.44 and 10 to the -2 and once we take the normal cdf inverse of that we can get the equivalent beta value in SORM and that gives us about 2.186.

Now this is interesting because we had in FORM 2.225 which we knew actually was an overestimation of reliability. So, now we have reduced the reliability to about 2.186 the index and now if we compare with the value obtained from Monte Carlo simulations about 2.180. So this value from SORM is much closer much closer to the to the true value obtained from MCS. Now just one point to note is that SORM in this case still overestimates reliability very slightly.

So that green curvature does not really mean that we are including the entire probability content

in in that green region the failure region demarcated by the green line because there were already approximations involved in computing that P_f , 2. Also but still in this example we have a much improved and almost correct value of the reliability index obtained through SORM.